the general science journal

NOETHER's THEOREM

From Standpoint of Physics

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Keywords: Noether's theorem, symmetry, energy conservation

Abstract: Is symmetry the cause of energy conservation?

1. Abstract

In present article I would like to take a brief look at Noether's theorem. There are two different opinions in the scientific world regarding this theorem. Some part of community claims that the theorem is nothing but tautology (Miles Mathis for example), while the mainstream science keep telling us that it is a greatest discovery ever made by mankind. Indeed, assuming such a simple property like symmetry one was able to deduce all conservation laws of physics. In my opinion this theorem is a very good example of high level mathematical abstraction. Nobody even think what kind of physical processes are laying underneath. I am going to make this exercise and take a look at the theorem from the solid platform of physics.

2. The Theorem (Discovery #1)

Does anyone ever read this theorem? I did not read it myself before. I read it only recently when I decided to see how this theorem is connected to the real world of physics.

Everybody hear that such a simple thing like the symmetry of our world leads to the laws of conservation. In particular the symmetry of space leads to momentum conservation, angular symmetry leads to angular momentum conservation and the symmetry of time leads to energy conservation. In this short article I would like to discuss just energy conservation.

Reading the theorem one could make the first discovery – the term "symmetry" is not related to our physical world. The symmetry is related to such mathematical thing as Lagrangian function.

This is our first discovery – nothing related to the physical world.

3. Symmetry (Discovery #2)

What the symmetry is? I expect that anyone familiar with geometry will start to tell us about scaling, reflection, rotation and of course beauty and harmony. But what does it mean – symmetry time? Should I change the sign of time in some formulas?

This is all wrong. We are talking about mathematics here. «Lagrangian function is symmetric or invariant under changes $t \to t + \delta t$». In simple words if you put any time value in the formula, you will get the same answer. Or Lagrangian function should not depend on time. It is a ***constant***.

Here I could expect the following objection. Why it should be a constant? It could depend on coordinates, but not depend on time! Well, if body is moving then its coordinates changes with time according to some rule and therefore the Langangian will be changed in time. With Langangian depending on coordinates the body should stay still if it wants its Lagrangian not to be time dependent. I am thinking that in the case everything stay still, I don't even need the theorem. Kinetic energy is zero and could not be changed; otherwise it is not a "still".

So, it is a constant. $\pi = 3.14\ldots$ is a constant – it is a symmetric in time.

And we just made our second "discovery" – it is nothing to do with symmetry, it is just mathematical synonym for the word "constant".

4. Lagrangian

What is Lagrangian function equals to? The answer is quite simple. The Langangian function is equals to kinetic energy minus potential energy.

$$L = T - V = const \qquad (1)$$

Next our question will be: under what condition Lagrangian function will remains constant? Let's write down the full energy of the system:

$$E = T + V = const \qquad (2)$$

We know from physics, that full energy conserves. Let now the potential energy of the system be reduced by the value of ΔE. The kinetic energy should be increased by exact same value. But what will happen to Lagrangian?

$$L = T + \Delta E - (V - \Delta E) = T - V + 2\Delta E = const$$

One could see that in order to keep Lagrangian function out of any change, ΔE should be equals to zero.

5. Conclusion

Keep in mind what we are just found, the Noether's theorem could be rephrased for the case of "time symmetry".

"If there are transformations between kinetic and potential energies then the full energy of the system will be conserved."

Or even like this: "If the kinetic energy of the system is not changing and potential energy is not changing then the total energy of the system will be conserved".

Is it tautology or not?

It's your call.

References: [1] Noether's theorem. Wikipedia. https://en.wikipedia.org/wiki/Noether%27s_theorem