

Surprises in spherical geometry

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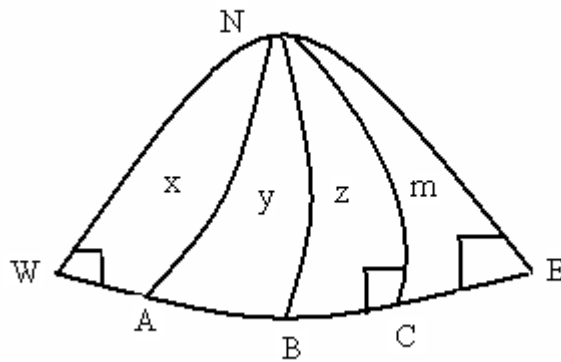
Abstract

By applying the fundamental operations of number theory to spherical triangle interior angle sum properties, a number of algebraic equations were formulated. The analysis of those equations yielded an interesting finding

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Construction



Spherical Figure 1

Let N be the north pole of a sphere and WE is its equator. AN, BN, CN, EN and WN are perpendiculars (great circle) to WE . x, y, z and m respectively denote the sum of the interior angles of spherical triangles NWA, NAB, NBC and NCE .

Results

Since angles WAB , ABC , and BCE are straight angles they are all each equal to 180 degree.

Let v be the value of this 180 degree. (1)

Let angle WNB = s , ANC = t and WBE = u (2)

Assuming (1) and (2) and adding we get that,

$$x + y = 2v + s \quad (3)$$

$$y + z = 2v + t \quad (4)$$

$$z + m = 2v + u \quad (5)$$

$$(3) - (4) \text{ gives , } x + t = z + s \quad (6)$$

$$(4) - (5) \text{ yields, } y + u = m + t \quad (7)$$

Squaring (3)

$$x^2 + y^2 + 2xy = 4v^2 + s^2 + 4vs \quad (3a)$$

$$\text{Squaring (7) } m^2 + t^2 + 2mt = y^2 + u^2 + 2yu \quad (7a)$$

Adding (3a) and (7a) we get that,

$$(x + u)(x - u) + (m + 2v)(m - 2v) + (t + s)(t - s) \\ + 2y(x - u) + 2mt - 4vs = 0$$

$$\text{i. e } (x - u)[x + u + 2y] + (m + 2v)(m - 2v) + \\ (t + s)(t - s) + 2mt - 4vs = 0$$

Replacing $x + y$ by $2v + s$, $y + u$ by $m + t$, $m - v$ by $u - z$ and $t - s$ by $z - x$ we have,

[See eqns. (3) , (5) , (6) and (7)]

$$(x - u) [2v + s + m + t] + (m + 2v) (u - z) + (t + s) (z - x) + 2mt - 4vs = 0$$

Rearranging, $(m + 2v) [x - u + u - z] + (t + s) [t - s + x - u] + 2mt - 4vs = 0$

i. e $(m + 2v) [x - z] + (t + s) [t - s + x - u] + 2mt - 4vs = 0$

Substituting $s - t$ for $x - z$ and $s + z$ for $x + t$ [See eqn.(6)] we have,

$$(m + 2v) [s - t] + (t + s) [z - u] + 2mt - 4vs = 0$$

i.e $t (z - u + 2m - m - 2v) + s (z - u - 4v - m - 2v) = 0$

Replacing $m + z$ by $2v + u$ [see eqn. (5)] we obtain, $2m + 2v = 0$

i. e $m + v = 0$, i.e $m = -v$ (8)

It is well known that in geometry minus theta represents the vertically opposite angles. Since vertically opposite angles are equal it implies

from (8) that $m = v$. (9)

Comparing (9) and (2) we get that the sum of the interior angles of spherical triangle NCE is equal to two right angles. (10)

Discussion

Needless to say, (10) establish the fifth Euclidean postulate which is one of the most famous mathematical impossibility. More over, the interior angle sum of a spherical triangle is more than 180 degrees. In the spherical figure the interior angle sum of triangle NCE is 180 degrees plus (+) angle CNE. There are no flaws in the algebra. So, there is a new hidden mathematical concept. Only further studies will unlock the concealed truth.

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