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Relativistic Time Dilation and the Muon Experiment

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Abstract

The time dilation of Lorentz-Einstein can be readily derived from the classical light clock experiment where the clock is positioned perpendicularly to the direction of its motion. The extent of dilation is given by the Lorentz factor: $1/\sqrt{1-v^2/c^2}$ where v is the relative velocity of the light clock and c is the speed of light.

This type of time dilation is apparently consistent with the muon experiment. Assuming that the Lorentz-Einstein time dilation is relevant to the light clock experiment when the clock is aligned along the direction of motion, the Lorentz-Einstein length contraction is usually then derived. However, if there is no such contraction we then deal with a time dilation of non-Lorentz-Einstein type. The amount of time dilation is now specified by the squared Lorentz factor: $1/1-v^2/c^2$. It appears that this type of time dilation is even much more agreeable with the muon experimental measurements than the Lorentz-Einstein type.

Keywords: Special relativity, Lorentz-Einstein, muon, time dilation, time contraction

1 Introduction

Time dilation and length contraction are two important effects of the Special relativity (SR). These two effects depend upon the second postulate of this theory that the speed of light c is the same in all inertial frames of reference [1, 2]. In general, the Lorentz-Einstein (LE) time dilation is a prerequisite for the LE length contraction which only occurs in the direction of motion of moving frame [1, 2].

To date several indirect LE length contraction experiments were performed [3 – 9] but no variation in length measured. Sherwin [10] has reported only one direct experiment and concluded that it was contradictory to the LE length contraction. As far as we are aware there is no critical evaluation and possible revision of his experiment. For many researchers the LE time dilation is indirectly demonstrated by - muon experiment [1]. The most of information about this experiment presented in this note is taken from the classic work by Frisch and Smith [7].

According to SR, the proper time ΔT_0 is a time interval of an event measured by an observer in an inertial frame stationary relative to the event. Moreover, the time interval of the same event has a longer duration ΔT, so called-improper time, as measured by an observer in an inertial frame moving with a relative speed v to the same event.

The proper length in SR is the length of an object in its rest frame. The improper length is the length of the object in any other frame where it is moving with a relative speed v.

Many physics textbooks that deal with the concepts of LE time dilation and length contraction depict using a device known as a light clock [1, 2]. In this note, we will especially focus our atention on the time dilation and length contraction when the light clock is positioned along the direction of motion. Of course, you may find all following derivations in many elementary physics texts.

2 Discusion and Conclusions

The light clock consists of two plane parallel mirrors M_1 and M_2 facing each other at a distance d apart as in Fig. 1a. I will here consider only the time intervals while a photon moving between mirror M_1 and mirror M_2. The lower mirror M_1 has a light source at the center that emits a photon (or a light signal) at 90 degrees in the direction of mirror M_2. Here it is usually assumed that the photon reaching the mirror M_2 is back-reflected to the mirror M_1. Taking into account this assumption, for the light clock at rest the proper time interval is $\Delta T_0 = 2d/c$. However, we must stress here that any of photon detections at the mirror M_2 would involves its annihilation.

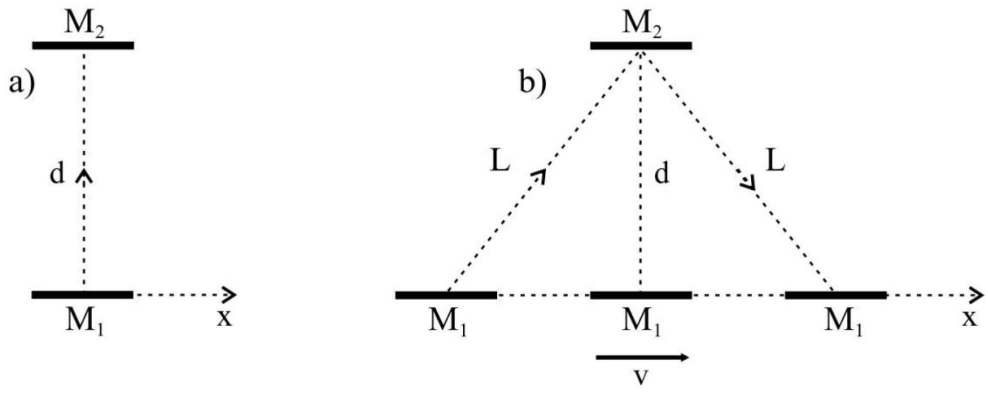

Fig. 1. Measurements and analysis made in different frames. The light clock positioned perpendicular to the x-axis. (a): No relative motion. (b): light clock moving at speed v.

Now allow the same light clock to be moving with a certain relative speed $\boldsymbol{v}$ horizontally in the direction of positive x-axis. An observer in the rest frame who is watching this clock could design a following diagram**,** Fig. 1b. Clearly, a photon will now travel the larger distance L and thus it will take a longer (improper) time

$$\Delta T = 2L/c$$

According to the LE transformation ΔT_0 and ΔT are related by the time dilation expression

$$\Delta T = \Delta T_0/\sqrt{1-v^2/c^2}$$

where $1/\sqrt{1-v^2/c^2}$ is the Lorentz factor or time dilation factor.

Let us now performed the experiments using the same light clock as in Fig 1a but now aligned along the x-axis, as shown in Fig. 2a. Obviously, if this clock is resting the time interval is

$$\Delta T_0 = 2d/c$$

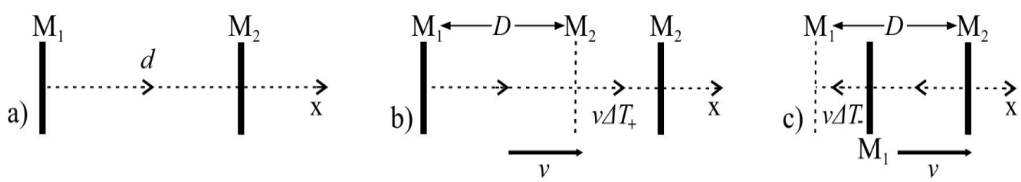

Fig. 2. Measurements and analysis for the light clock positioned along the x-axis. (a): No relative motion; (b) and (c): the light-clock moving at speed v.

Allow now this clock to move in the direction of the positive x-axis with the speed v, Fig. 2b. To reach now the mirror M_2 the photon will travel the distance

$$c\Delta T_+ = D + v\Delta T_+$$

in the time interval

$$\Delta T_+ = D/(c - v)$$

where D is the length of light clock measured by a stationary observer.

For the "back-reflected" photon the time interval

$$\Delta T_- = D/(c + v)$$

The total time needed for photon to complete cycle

$$\Delta T = \Delta T_+ + \Delta T_- = 2cD/(c^2 - v^2)$$

Factoring out c^2 in the denominator

$$\Delta T = 2D/c(1 - v^2/c^2)$$

To acquire a relationship between D and d one then invokes the LE time dilation $\Delta T = \Delta T_0 /\sqrt{1-v^2/c^2}$ and then write

$$2D/c(1 - v^2/c^2) = 2d/c\sqrt{1 - v^2/c^2}$$

After simplification, we get the LE length contraction

$$D = d\sqrt{1 - v^2/c^2}$$

In this derivation we hypothesize a priori that the length of the moving light clock is (of course, relativistically) different than the proper length: $d \neq D$ and the time of this clock is "dilated" relative to the proper time ΔT_0 according to the LE transformation: $\Delta T = \Delta T_0 /\sqrt{1-v^2/c^2}$. However, there is no experimental evidence corroborating this hypothesis.

Let us assume first that there is no relativistic length contraction i. e., $D = d$ then

$$\Delta T = \Delta T_0/(1 - v^2/c^2)$$

Thus, there is also time dilation but with the squared Lorentz factor $1/(1 - v^2/c^2)$ instead $1/\sqrt{(1 - v^2/c^2)}$ predicted by LE (hereinafter: the non-LE time dilation). It remains to find out whether the non-LE time dilation agrees with the experiments.

The most convincing experimental evidence for the time dilation in the direction of the motion comes from the cosmic muon experiments. Muons are created in the upper Earth's atmosphere are secondary products of interactions between primary cosmic rays and the nuclei of atmospheric molecules. Muons travel at relativistic speeds and are unstable particles with a mean lifetime at rest $T_0 \sim 2.2$ μs. At the relativistic speeds the muons experience time dilation. This dilation allows them to reach the Earth's surface before they decay. To measuure time dilation, the mean lifetime of muons at rest are compared with the apparent increase in the lifetime of muons in motion using the measurements of their speeds (v/c) and the changes of muon flux with altitude.

The original muon experiment was first done by Rossi & Hall (6) in 1941 who measured muon fluxes at the top of Mt Washington (New England, USA) about 2 km high, and at the base of the mountain. Their experimental results were consistent with the relativitistic time dilation. The experiment has since been repeated by a number of other researchers.

Let us assume that the muons are moving vertically down from the height of 10 km at the mean speed of 0.98c (= 0.98 × 299,792 km s^{-1} = 293,796 km s^{-1}). Ignoring relativistic effects, they would travel t ~ 34 μs before reaching ground level. The fraction of the muons reaching the ground level is given by

$$P = 2^{-t/T_0} = 2^{-34/2.2} = 2.2 \times 10^{-3}\ \%$$

Thus, almost no muons could be expected to reach ground level.

In the light of the above, the muons moving at a relativistic speed could have a mean lifetime either $T_L = T_0 \times 1/\sqrt{1 - v^2/c^2}$ (LE time dilation) or $T_{nL} = T_0 \times 1/1 - v^2/c^2$ (the non-LE time dilation). Thus, at the mean speed of 0.98*c*, the mean lifetime of the muons T could be either $T_L \sim 11$ μs or $T_{nL} \sim 55$ μs. The fraction capable to reach ground level turns now into

$$P = 2^{-t/T}$$

For the lifetime of $T_L \sim 11$ μs and $T_{nL} \sim 55$ μs this fraction would be about 12 % and 65 %, respectively.[1] Hence, a much larger fraction of the muons (generated in the upper atmosphere) would be capable of reaching ground level in the non-LE case than in the LE. Besides, this much higher prediction is even more consistent with experimental measurements. The question is now how to decide which of these two time dilations is right?

Let us assume now that there is no time dilation for the moving light clock oriented along the direction of motion (i. e., $\Delta T = \Delta T_0$). Then it is easy to show that $D = d\,(1 - v^2/c^2)$. Thus, there is a length contraction by the non-Lorentz factor.

In conclusion, from the experiment of moving light clock oriented peprendicular to the direction of motion the demonstration of LE time dilation is simple and straightforward. However, there is no way to illustrate the coexisting LE length contraction. On the other hand, there is no a simple and straightforward way to demonstrate the LE time dilation or length contraction in the case of moving light clock positioned along the direction of motion without assuming a priori the length change according to the LE transformation. However, for this assumption there is no experimental evidence.

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[1] In previous version of this communication the author erroneously wrote respectfully instead respectively. Please, accept his apology.

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