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The Minimum Speed of a Free Non-Relativistic Massive Particle and Its Uncertainty in Position: a Brief Note

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The simplest form of Heisenberg's uncertainty principle is:

$$\Delta p \Delta x \geq h \quad \ldots (1)$$

where Δp is the uncertainty in the momentum of a particle and Δx is its uncertainty in position both in the same direction, and h (= 6.63×10^{-34} J sec) is Planck's constant. In other words, it is impossible simultaneously to know the momentum and position of a particle with infinite accuracy.

Once the uncertainty in the momentum Δp is found, the uncertainty in speed $\Delta \upsilon$ can be found from:

$$\Delta p = m\Delta \upsilon$$

and vice versa. Of course, this equation is valid for a free non-relativistic massive particle (FNMP) with a mass at rest m_0:

$$\Delta p = m_0\Delta \upsilon \ \ldots (2).$$

Premović [1] has derived the following equation for the minimum speed of this particle within the non-relativistic framework:

$$\upsilon_{min} = h/m_0D_0 \quad \ldots (3).$$ [1]

In this derivation, he followed the analogy with light and assumed (rather reasonably) that this particle is spherical and has a diameter at rest, D_0. Assuming the minimum speed must be at least equal to its uncertainty then $\Delta \upsilon$ for the FNMP must be equal or larger than υ_{min}. So, eqn. (2) has the following form:

$$\Delta p \geq m_0\upsilon_{min}.$$ [2]

The minimum uncertainty in the momentum for a given FNMP is then:

[1] This speed is a constant characteristic of a given FNMP [1].
[2] This is a constant characteristic of a given FNMP.

$$\Delta p_{min} = m_0\upsilon_{min.}$$

Combining this equation and equation (3) and after a bit of algebra, we have:

$$\Delta p_{min} = h/D_0.$$[3]

In other words, the minimum uncertainty in the momentum of a (reasonably spherical) free non-relativistic massive particle equals to Planck's constant divided by its diameter. This is in direct contradiction with Heisenberg's uncertainty principle which allows for an FNMP an infinitely small uncertainty in momentum [2, and references therein].

For an FNMP **exp.** (1) can be written as follows:

$$m_0\upsilon_{min}\Delta x \geq h.$$

The minimum of this product is

$$m_0\upsilon_{min}\Delta x = h.$$

This speed is constant and characteristic of a given FNMP [1].

Combining this equation and equation (3) and after again a bit of calculus, we get the maximum position of an FNMP:

$$\Delta x = D_0.$$[4]

In other words, the maximum uncertainty in the position of a (reasonably spherical) non-relativistic free massive particle equals the diameter of this particle. This is also in direct contradiction with Heisenberg's uncertainty principle which allows for this particle an infinitely large uncertainty in position [2, and references therein].

Since eqn. (3) is also valid for a free relativistic massive particle [1] this conclusion can be applied to this particle. However, as far as this author is aware the incompatibility between Special relativity and the Uncertainty principle is still a problem of modern physics that requires a sound solution.

References

[1] P. I. Premović, *The Minimum speed of a free massive particle*. The General Science Journal, December 2021.

[2] V. Todorinov, P. Bosso, S. Das, *Relativistic generalized Uncertainty Principle*. Annals Phys. 405, 92-100 (2019).

[3] This is a constant characteristic of the FNMP in question.