The Dimensionality of the Geiger Nuttall Law: a simple but interesting note

Pavle I. Premović
Laboratory for Geochemistry, Cosmochemistry and Astrochemistry
e-mail: pavleipremovic@yahoo.com

Abstract. It appears that the natural logarithm form of the empirical Geiger-Nuttall law is not dimensionally correct. It is suggested a new type of this form as well four new non-exponential forms of this law.

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One of the milestones of quantum mechanics was the formulation of the Geiger-Nuttall law [1, 2]. This law states that there is a simple empirical relation between the α-decay energy $Q_\alpha$ and the half-life of heavy alpha (α)-emitter $t_{1/2}$

$$\log t_{1/2} = A + B/\sqrt{Q_\alpha}$$

where $A$ and $B$ are constants. Let us convert this equation into a natural logarithm (LN) form

$$\ln t_{1/2} = A + B/\sqrt{Q_\alpha}$$

where $a = 2.303A$ and $b = 2.303B$.\(^1\) The kinetic energy, $E_\alpha$, of the emitted α-particle is slightly less (about 0.4 % for heavy α-emitters, such as the uranium α-emitters) than $Q_\alpha$. Therefore, the last equation can be written as follows

$$\ln t_{1/2} = a + b/\sqrt{E_\alpha} \quad \ldots \ (1).$$

After the conversion of this equation into the exponential form we have

$$t_{1/2} = e^{a + b/\sqrt{E_\alpha}} \quad \ldots \ (2).$$

Obviously, the dimension\(^2\) of the left side of this expression is T but its right side is dimensionless. Hence, eqn. (2) is dimensionally incorrect. The simplest way to make this equation dimensionally correct is to multiply its right side with the constant $\theta$ equals or close to 1 and expressed in time unit of $t_{1/2}$ (e. g. seconds)

$$t_{1/2} = e^{a + b/\sqrt{E_\alpha}} \times \theta$$

\(^1\) $\ln t_{1/2} = 2.303\log t_{1/2}$
\(^2\) Just to remind the reader that M, L and T are the symbols for basic dimensions: mass, length and time, respectively. Of note: in previous version of this communication the author erroneously wrote respectfully instead respectively. Please, accept his apology.
Since lnθ is equal or close to 0 the LN form of this equation is identical to the initial eqn. (1) of the Geiger-Nuttall law.

There is another way to resolve the above dimensionality issue. Instead the eqn. (1) derived from empirical Geiger-Nuttall law we may propose the following equation

\[ t_{1/2} = \frac{ab}{\sqrt{E_{a}}} \]  

(4).

The LN form of this equation is \( \ln t_{1/2} = \ln a + \ln \left( \frac{b}{\sqrt{E_{a}}} \right) \) where \( a \) and \( b \) are the constants. The last equation is a linear form of \( t_{1/2} \) vs. \( 1/\sqrt{E_{a}} \). After some rearrangement of eqn. (4) we get

\[ t_{1/2} = a\sqrt{\frac{b^2}{E_{a}}}. \]

Keeping in the mind that the dimension of \( E_{a} \) is \( ML^2T^{-2} \), a dimensional analysis shows that this equation is dimensionally correct for the fourth following cases: 1) if the dimension of \( a \) is \( L \) then the dimension of \( b^2 \) must be \( M \); 2) if the dimension of \( a \) is \( T \) then the dimension of \( b^2 \) must be \( ML^2T^{-2} \), i.e. \( \frac{b^2}{E_{a}} \) must be dimensionless; 3) if the dimension of \( a \) is \( M \) then the dimension of \( b^2 \) must be \( M^{1}L^{2} \); and finally 4) if \( a \) is dimensionless then the dimension of \( b^2 \) must be \( ML^2 \).

The detailed analysis of each of the above cases, including eqns. (1) and (2), will be subject of author’s further study.

Recently, Premović [3] derived a new mathematical form for the Geiger-Nuttall law. This is expressed by the following equation

\[ t_{1/2} = u \times w \times \frac{1}{\sqrt{E_{a}}} \]

where \( u \) and \( w \) are the constants. According to Premović [7], the dimension of \( u \) is \( L \) (analogous the above case 1) and the dimension of \( (w \times 1/\sqrt{E_{a}}) \) is \( [L^{-1} \times T] \) or the dimension of \( w \) is \( M \) Therefore, the left side of this expression is dimensionally consistent with its right side.

References


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3 Since \( E_{a} = 1/2mv^2 \) (where \( m \) is the mass of the α-particle and \( v \) is its speed) and the dimension of \( v \) is \( LT^{-1} \) then the dimension of \( E_{a} \) is \( ML^2T^{-2} \)