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The Lifetimes of ^{238}U Nuclei, the Escaping Attempts of Their Alpha Particles and the Geiger-Nuttall Law

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Abstract. We consider here the large populations of ^{238}U nuclei and their emitted alpha particles. The main points are: (a) a time interval for which alpha particle of ^{238}U nucleus makes one attempt to escape is approximately 10^{-21} second; (b) the number of the mean escape attempts is ~ 2×10^{38}; (c) contribution of the tunneling time of ^{238}U nucleus to the mean time and to the lifetime of each individual nuclei is negligible; (d) the sum of the lifetimes of ^{238}U nuclei and the sum of the number of escape attempts of their alpha particles are constant. Consequently, the ratio of these two sums is also constant, and is about 10^{-21} seconds; and, (e) a new mathematical form of the Geiger-Nuttall law is derived using a very simple theoretical approach.

Keywords: Radioactive decay, mean lifetime, ^{238}U, α-particle, escape, Geiger-Nuttall law.

Introduction. Radioactive decay is a purely statistical (random) process. If the number of radioactive nuclei is large enough then the total number of radioactive nuclei N_t, that have not yet been transformed at a time t, can be obtained from the following equation

$$N_t = N_0e^{-\lambda t} \quad \dots (1)$$

where N_0 is the number of radioactive nuclei at a time $t = 0$ and λ is the decay constant. This equation is the final mathematical representation of the radioactive decay law, and it applies to all decays. The decay pattern of a large number of radioactive nuclei is easy to predict using eqn. (1). For practical purposes, we will be using Avogadro's number $\mathcal{N}$ (= 6.022×10^{23}) as a reasonable approximation for a large number of radioactive nuclei.

In general, there are two ways to measure the time for which a radioactive nucleus is stable: half-life $t_{1/2}$ and the mean lifetime τ. By the definition of the term half-life, when $N_t = 1/2N_0$ then $t = t_{½}$. In accordance with the definition of the term half-life, when $N_t = 1/2N_0$ then $t = t_{½}$. A simple algebra of eqn. (1) shows that $t_{1/2} = (\ln 2)/\lambda$.

The quantity τ is the reciprocal of the decay constant λ or $\tau = 1/\lambda$; τ is usually estimated from the measured $t_{1/2}$ using the following relation $\tau = 1.44t_{1/2}$. The author has some reservations about these equations because they are derived by combining the discrete nature of the number of radioactive nuclei and the continuous exponential approximation. He will discuss this question in one of his next communications. Of course, λ, $t_{1/2}$ and τ have only meaning for a large population of radioactive nuclei (of the same type, of course).

We will concentrate on the lifetimes of a large population of the decayed ^{238}U nuclei, as well as on the escaping times of their emitted α-particles.

Results and Discussion. If there is an initial population of $\mathcal{N}$ radioactive, the sum of their all individual lifetimes divided by $\mathcal{N}$ is the mean lifetime τ. mathematically speaking, τ is the arithmetic mean of lifetimes of all $\mathcal{N}$ individual nuclei

$$\tau = (\tau_1 + \tau_2 + ... + \tau_{\mathcal{N}-1} + \tau_{\mathcal{N}})/\mathcal{N}$$

where i and τ_i denote, respectively, individual radioactive nuclei and their lifetimes.
Rearranging this equation gives

$$\mathcal{N}\tau = (\tau_1 + \tau_2 + ... + \tau_{\mathcal{N}-1} + \tau_{\mathcal{N}}) \quad \ldots (2).$$

After a bit of algebra and estimation, we found that about 62.5 % of τ_i is smaller than $1.44t_{1/2}$ or τ. Therefore, for a large population of radioactive nuclei about 37.5 % of τ_i exceeds τ.

Consider alpha (α-) decay of (parent) ^{238}U (initially at rest) with τ of about 2×10^{17} second to (daughter) ^{234}Th. It is assumed that the parent ^{238}U nucleus before decay consists of the daughter ^{234}Th and α-particle. This particle is in a free (non-bound) state, otherwise, the decay could not occur. The emitted α-particle whose mass m_α is ca. 6.65×10^{-27} kg has a speed $v_\alpha = 1.42\times10^7$ m sec^{-1}. This speed is determined from its non-relativistic kinetic energy $E_\alpha = 1/2\ m_\alpha v_\alpha^2$, ca. 4.2 MeV. In fact, v_α is its speed when α-particle escapes from the ^{238}U nucleus.

According to classic mechanics, α-particle with the E_α of ca. 4.2 MeV could never be able to overcome the Coulomb potential barrier of ^{238}U, V_C, which is probably about 20 MeV [1] and to escape the nucleus. Quantum mechanics provides an explanation based upon the concept of tunneling where this particle can be found in a classically forbidden (outside) region of the nucleus.

The size (diameter) d of the nucleus of the heaviest atoms, such as uranium (U), is about 15×10^{-15} m or 15 fm. In fact, an α-particle must make many attempts back and forth across ^{238}U nucleus before it can escape; in fact, this particle oscillates along the diameter of nucleus. A time interval for which the α-particle of ^{238}U nucleus makes one "try" to escape is approximately given by $d/v_\alpha = 10^{-21}$ seconds. In other words, α-particle effectively attempts to tunnel through the Coulomb barrier $v_\alpha/d = 10^{21}$ times per second. Denote now with n and n_i, respectively, the mean escape attempts and the individual escape attempts of the α-particle before leaving ^{238}U nucleus. Let also t(e) and $t_i(e)$ stand for the mean escaping time and the escaping time, respectively. Simple reasoning indicates that $t(e) \sim \tau$ and $t_i(e) \sim \tau_i$ because the tunneling time T could contribute to these escaping times.

A question that goes along with tunneling is: how long it takes for α-particle to tunnel the Coulomb barrier. The tunneling time problem is one of the long-standing and controversial problems in quantum mechanics. There are numerous attempts to solve it but none of them gives a flawless answer to this question [2].

The width of the barrier w a through which an α-particle must tunnel can be roughly calculated using the following equation: $w = (V_C/E_\alpha)d - d$ [1]. Inserting in this equation the values for V_C, E_α and d we obtain w ~ 57 fm which is probably much smaller about 30 fm [1]. Of course, the speed of this particle during the tunneling process cannot exceed the speed of light c, therefore, the lower limit for the tunneling time $T = w/c \sim 1 \times 10^{-22}$ second; its upper limit $w/v_\alpha \sim 1.5\times10^{-21}$ second. Therefore, contribution of the tunneling time to τ is negligible. Consequently, $t(e) \sim \tau = nd/v_\alpha$ then the number of the mean escape attempts

$$n = \tau v_\alpha/d \quad \ldots (3).$$

We know that τ is about 2×10^{17} sec and $v_\alpha = 1.42\times10^7$ m sec^{-1} for ^{238}U so $n \sim 2 \times 10^{38}$ is the constant characteristic for this uranium isotope. In general, n of eqn. (3) is the constant characteristic for any isotope α-emitter of the heaviest atom (including any uranium isotope α-emitter, of course). Since T contributes to each then $t_i(e) = n_i d/v_\alpha + T = 10^{-21}n_i + T$. Of course, the maximum contribution of T is about 1.5×10^{-21} second and the maximum $\tau_i = 10^{-21}n_i + 1.5 \times 10^{-21}$ second. For $n_i \geq 100$ we can practically take that $\tau_i = 10^{-21}n_i$. By inserting this τ_i into eqn. (2) we obtain

$$\mathcal{N}\tau = 10^{-21}(n_1 + n_2 + \ldots + n_{\mathcal{N}-1} + n_{\mathcal{N}}) \quad \ldots (4).$$

Plugging the values for $\mathcal{N}$ and τ into the equations (2) and (4) we get

$$\tau_1 + \tau_2 + \ldots + \tau_{\mathcal{N}-1} + \tau_{\mathcal{N}} = 1.2\times10^{41} \text{ second}$$

and

$$(n_1 + n_2 + \ldots + n_{\mathcal{N}-1} + n_{\mathcal{N}}) = 1.2\times10^{62}.$$

Thus, the sums of lifetimes of a large population of ^{238}U nuclei and the number of escape attempts of their α-particles are fixed. A similar conclusion is true for other appropriate radioactive isotopes.

If the number of decayed ^{238}U nuclei is k times greater or smaller (but still large) then the above sums would be k times higher or lower. However, the ratio of eqns. (2) and (4) is constant and equals: $\tau/n = \tau_i/n_i = 10^{-21}$ second even if $k \to \infty$, of course. This is of some importance. Any single ^{238}U nucleus which we observe is not all alone in the Universe and it belongs to an infinitely large total number ($k \to \infty$) of its ^{238}U nuclei. Therefore, the ratio τ/n of ^{238}U of about 10^{-21} seconds is one of the "magic" units of time of the Universe.

Geiger-Nuttall law [3, 4] is a semi-empirical law that expresses the half-life of a heavy α-emitter in terms of the kinetic energy of its released α-particle. In its modern natural logarithm (LN) form this law is: $\ln t_{1/2} = a + b/\sqrt{Q_\alpha}$, $\ln t_{1/2} = a + b/\sqrt{Q_\alpha}$ where Q_α is the α-decay energy a and b are the constants that can be determined by fitting to experimental data for each isotopic series. The kinetic energy, E_α, of the emitted α-particle is rather slightly less than Q_α.[1] Therefore, the above

[1] The kinetic energy of the recoiling ^{234}Th nucleus produced in the decay of ^{238}U is 0.070 MeV.

equation can be rewritten as $\ln t_{1/2} = a + b/\sqrt{E_\alpha}$. The quantum tunneling enables one to obtain the Geiger-Nuttall law, including coefficients, via direct calculation.

We know that $n = \tau v_\alpha/d$ and $v_\alpha = \sqrt{2E_\alpha/m_\alpha}$. Combining these two equations and after a bit of algebra, we get $\tau = nd \times \sqrt{(m_\alpha/2)}/\sqrt{E_\alpha}$. Substituting in this equation $1.44t_{1/2}$ instead τ, u instead (nd/1.44) and w instead $\sqrt{(m_\alpha/2)}$ we obtain

$$t_{1/2} = u \times w \times \sqrt{1}/\sqrt{E_\alpha} \quad \ldots (5)$$

where u and w are the constants. Its LN form is

$$\ln t_{1/2} = \ln u + \ln w + \ln(1/\sqrt{E_\alpha}) \quad \ldots (6).$$

Evidently, u and w are also the constants. This equation is a linear function of $\ln t_{1/2}$ vs. $\ln(1/\sqrt{E_\alpha})$. To illustrate this, we will use some experimental data for uranium isotopes ^{228}U – ^{238}U given in Table 1. The plot $\ln t_{1/2}$ vs. $\ln(1/\sqrt{E_\alpha})$ for these isotopes gives a straight line, Fig. 1. Of course, it is before necessary to convert seconds (sec) to years (yr) and joules (J) into MeV of this equation.

Uranium isotope	E_α [MeV]	$t_{1/2}$
238	4.198	4.468×10^9 yr
236	4.494	2.342×10^7 yr
235	4.395	7.037×10^8 yr
234	4.775	2.455×10^5 yr
232	5.320	69.8 yr
230	5.888	20.8 day
228	6.680	9.1 min
226	7.570	0.35 sec

Table. 1 Some experimental data of uranium isotopes ^{226}U – ^{238}U [5-8].

It appears that the above new form of the semi-empirical Geiger-Nuttall law represents until now the simplest theoretical way to derive this law. It is also possibly applicable to other α-emitters of the heaviest atom isotopes.

We know that $m_\alpha = 6.65 \times 10^{-27}$, 1 yr = 3.15×10^7 sec and that 1 J = 6.24×10^{12} MeV. After a simple algebra the eqn. (5) can be written as follows

$$t_{1/2} = nd/1.44 \times (3.15 \times 10^7) \times \sqrt{[(6.25 \times 10^{12}) \times m_\alpha/2](1/\sqrt{E_\alpha})} \quad \ldots (7).$$

In the case of ^{238}U, for example, plugging into this equation the values for n (= 2×10^{-38}), d (= 15×10^{-15} m), m_α (= 6.65×10^{-27}kg) and E_α (= 4.198 MeV) we calculate $t_{1/2} \sim 4.74 \times 10^9$ years. The experimental value for $t_{1/2}$ is ~ 4.47×10^9 years, Table 1. Our theoretical approach is therefore not "too" bad. A detailed consideration of eqn. (7) will be a subject of the next author's study.
For the sake of clarity, let us consider briefly u and w of eqn. (5). According to eqn. (7), the constant u is a product of two variable terms: n and d and the constant w consist of three fixed

terms: m_α and two conversion factors J into MeV (6.25×10^{12}) and sec into yr (3.15×10^7). Therefore in this equation u (= nd/1.44) is the constant which is characteristic for each of the isotope α-emitters of the heaviest atoms and w [$= 1/(3.15 \times 10^7] \times \sqrt{(6.25 \times 10^{12} \times m_\alpha/2)}$] is the general constant, the same for all of these α-emitters. In contrast, the a and b constants in the original mathematical form of the Geiger-Nuttall are characteristic for each isotopic series.

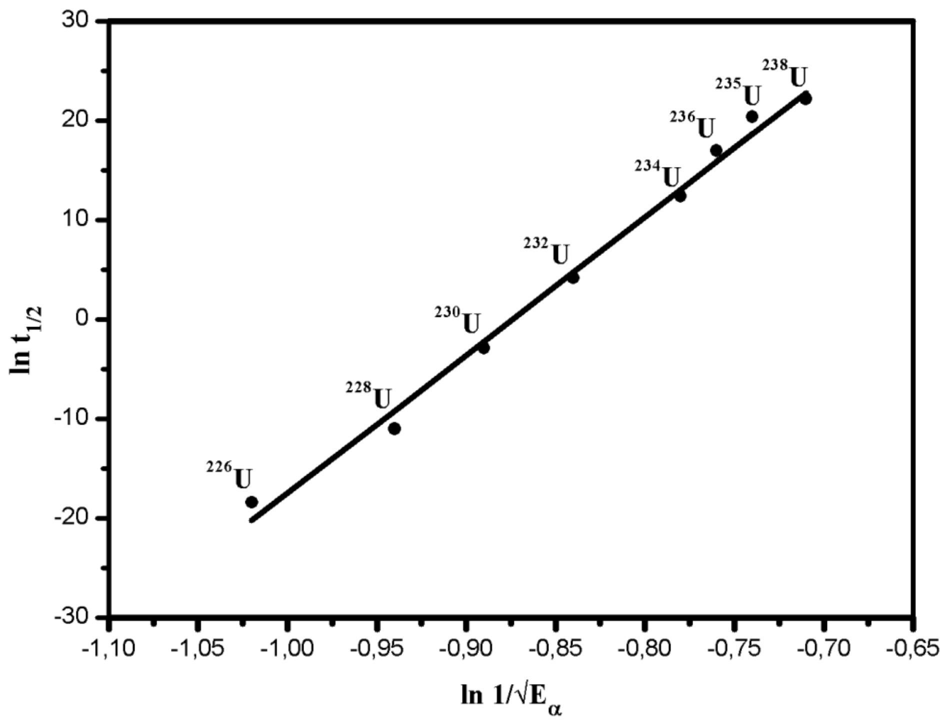

Figure 1. A comparison of the eqn. (6) with experimental data for ^{226}U - ^{238}U [5-8]. The linear fit to the values is shown by the solid line.

There are some intriguing details related to α-particles inside ^{238}U nucleus. Making the mean escape attempts n (~ 2×10^{38}) α-particle has to travel a distance: $2\times10^{38}\times d \sim 2 \times 10^{38} \times 15\times \times 10^{-15}$ m ~ 3×10^{21} km. The light-year is about ≈ 10^{13} km. As a result, on average, α-particle has to travel the distance of about 300 millions of light-years before escaping ^{238}U nucleus. Of note, our galaxy Milky Way is just about 100 thousands of light-years across.

It is suggested that a part of the Earth's ^{238}U atoms is formed in a supernovae explosion about 6 billion years ago. Let us suppose that at that time there were two "neighboring" nuclei n_s and n_e of ^{238}U nuclei and the nucleus n_s disintegrated at that time emitting α-particle. Allow right now that n_e decays and emits α-particle. This nucleus has made a "trip" of about 270 billion of light-years after the disintegration of n_s. Of note, the diameter of the observable universe is about 93 billion light-years. A question is why n_s needs so an incredible long "trip" and corresponding incredible time to escape ^{238}U nucleus? What is the cause of such 6 billion years delay? Moreover, if the decay happens right now why it did not occur before since there was a very long time to occur. These are some of the intriguing questions for which modern physics has not yet provided reasonable answers.

Conclusion. The main points are:

1. A time interval for which α-particle of ^{238}U nucleus makes one attempt to escape is approximately 10^{-21} second. The number of the mean escape attempts $n \sim 2\times10^{38}$.

2. Contribution of the tunneling time T to τ and τ_i of ^{238}U is negligible.

3. The sum of the lifetimes of ^{238}U these nuclei and the sum of the number of escape attempts of their α-particles are constant. Consequently, the ratio of these two sums is also constant, and is about 10^{-21} second.

4. A new expression for the Geiger-Nuttall law is reproduced on a very simple way.

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