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Are the Hydrogen Atom and Universe Energetically and Spatially Finite?

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Bohr's equation for the electron's energy of the hydrogen (H) atom is

$$E_n = -1/n^2(me^4/8\varepsilon_0^2 h^2) \quad \ldots (2)$$

where n (= 1, 2, 3,…) is the quantum number, m is the relativistic mass of the electron,

e (= 1.6×10^{-19} *C*) is its charge, ε_0 is the permittivity of vacuum (= 8.854×10^{-12} F m^{-1}) and *h* (= 6.63×10^{-34} *J sec*) is Planck's constant.[1]

According to this equation, the electron's energy of this atom in the ground state

$$E_1 = -\, me^4/8\varepsilon_0^2 h^2.$$

It is convenient to express the electron's energy in the n*th* orbit in terms of this energy, as

$$E_n = -E_1/n^2 \quad \ldots (2).$$

Combining eqns. (1) and (2), and after a bit of algebra, we get

$$E_n - E_1 = \Delta E_{1\rightarrow n} = -E_1(1 - 1/n^2) \ldots (3).$$

If $n \rightarrow \infty$, $\Delta E_{1\rightarrow n} \rightarrow -E_1$. $-E_1$ is the energy needed to remove the electron from the H atom and it is called the ionization energy = 13.6 *eV*. In contrast, eqn. (3) implies that the electron would never reach this energy and would not be removed as $(1 - 1/n^2) < 1$. However, it does. This implies that the value of n is finite (and cannot be infinite).

Elementary quantum physics combines this equation with Bohr's first quantization condition to solve for the radius of the electron's n*th* orbit

$$r_n = n^2[\varepsilon_0(h^2/m_0e^2) \quad \ldots (4).$$[2]

If $n \rightarrow \infty$, $r_n \rightarrow \infty$. As we stated above, the value of n can only be limited so the value of r_n. Therefore, this note suggests that the hydrogen atom is energetically and spatially finite. Does this also mean that the Universe is energetically and spatially finite?

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[1] It is often considered that the Schrödinger equation is superior to Bohr's equation in describing the H atom. In most cases, the results of both approaches coincide or are very close.

[2] The radius of the first Bohr orbit, denoted as a_0, is obtained by setting n = 1 in eqn. (4)
$a_0 = \varepsilon_0(h^2/m_0e^2)$.