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The Dimensionality of the Geiger-Nuttall Law: a simple but interesting note

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Abstract. The natural logarithm form of the empirical Geiger-Nuttall law is not dimensionally correct. A new type of this form and four new non-exponential forms of this law are suggested.

Keywords: Geiger-Nuttall law, quantum mechanics, dimensionality, alpha, emitter.

Derivations and Discussion. One of the milestones of quantum mechanics was formulating the Geiger-Nuttall law [1, 2]. This law states that there is a simple empirical relation between the α-decay energy Q_α and the half-life of heavy alpha (α)-emitter $t_{1/2}$

$$\log t_{1/2} = A + B/\sqrt{Q_\alpha}$$

where A and B are constants. Let us convert this equation into a natural logarithm (LN) form

$$\ln t_{1/2} = a + b/\sqrt{Q_\alpha}$$

where a = 2.303A and b = 2.303B.[1] The kinetic energy, E_α, of the emitted α-particle is slightly less (about 0.4 % for heavy α-emitters, such as the uranium α-emitters) than Q_α. Therefore, the last equation can be written as follows

$$\ln t_{1/2} = a + b/\sqrt{E_\alpha} \quad \ldots (1)$$

After the conversion of this equation into the exponential form we have

$$t_{1/2} = e^{a + b/\sqrt{E_\alpha}} \quad \ldots (2).$$

The dimension[2] of the left side of this expression is T but its right side is dimensionless. Hence, eqn. (2) is dimensionally incorrect. The simplest way to make this equation dimensionally correct is to multiply its right side with the constant θ expressed in time unit of $t_{1/2}$ (e.g. seconds)

$$t_{1/2} = e^{a + b/\sqrt{E\alpha}} \times \theta.$$

[1] $\ln t_{1/2} = 2.303 \log t_{1/2}$
[2] Just to remind the reader that M, L and T are the symbols for basic dimensions: mass, length and time, respectively.

Recently, Premović [3] derived a new mathematical form for the Geiger-Nuttall law. This is expressed by the following equation

$$t_{1/2} = u \times w \times 1/\sqrt{E_\alpha}$$

where u and w are the constants. According to Premović [1], the dimension of u is L and the dimension of ($w \times 1/\sqrt{E_\alpha}$) is [$L^{-1} \times T$] or the dimension of w is M Therefore, the left side of this expression is dimensionally consistent with its right side.

References

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[3] P. I. Premović, *The lifetimes of ^{238}U nuclei, the escaping attempts of their alpha particles and the Geiger-Nuttall law.* The General Science Journal, December 2021.