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The Finite Physical Quantities of the Bohr Model of Hydrogen Atom

Pavle I. Premović
Laboratory for Geochemistry, Cosmochemistry and Astrochemistry,
University of Niš, P.O. Box 224, 18000 Niš, Serbia

According to the Bohr model, the hydrogen atom consists an electron which orbits a proton The proton has a positive charge equal in magnitude to a unit of electron charge e (= 1.60×10^{-19} *C*) and its radius of about 0.85×10^{-15} *m*. The radius of the ground state orbit of hydrogen atom is symbolized as a_0 and it is about 5.3×10^{-11} *m*.

The energy expression for the Bohr model of hydrogen atom is

$$E_n = -2\pi^2 m_e e^4/n^2h^2$$

where:

m_e = the rest mass of electron
n (= 1, 2, 3,…) is a quantum number
h (= 6.63×10^{-34} *J sec*) is Planck's constant.[1]

We mark with E_1 the energy of an electron in the ground orbit of hydrogen atom with n = 1. This energy is – 2.18×10^{-18} J (or – 13.6 *eV*). The energies of all the following orbits successively with 1, 2, 3,... by their quantum number n and if $n \rightarrow \infty$, $E_n \rightarrow 0$.

The Coulomb force of attraction between the proton and electron of hydrogen atom is

$$F_p = k_e e^2/R_n^2$$

where k_e is the Coulomb constant (= $8{,}988 \times 10^9$ $N\,m^2\,C^{-2}$)[2] and R_n is the distance between these particles. This force causes the electron to deflect from a straight line to a circular orbit around the proton. This is centripetal force and it is balanced by a centrifugal force F_f given by the formula $m_e\upsilon_n^2/R_n$ where υ_n is the electron speed. In the equation form

$$F_p = F_f$$

or

$$m_e\upsilon_n^2/R_n = k_e e^2/R_n^2 \quad \ldots (1).$$

[1] Although the Bohr model of the hydrogen atom has many shortcomings, the energy equation of the Schroedinger quantum mechanical model for this atom gave the same energy equation as the Bohr equation. In both the Bohr model and quantum mechanics, the energy is proportional to $1/n^2$, according to eqn. (1).
[2] To avoid confusion in further text, the SI units are given in italics.

The Bohr quantum condition is defined by the following equation

$$m_e\upsilon_nR_n = n\hbar \quad \ldots (2)^3$$

where ħ (= $h/2\pi = 1.055 \times 10^{-34}$ *J sec*) is the reduced Planck constant. Combining this equation and eqn. (1) we get, after a bit of algebra,

$$\upsilon_n = k_ee^2/n\hbar.$$

This equation indicates that $\upsilon_n \leq k_ee^2/\hbar$, so its maximum value $\upsilon_1 = k_ee^2/\hbar$. Plugging into it the above known values we find $\upsilon_1 \approx 2.2 \times 10^6$ *m sec*$^{-1}$. Substituting $k_ee^2/\hbar$ of this equation with υ_1, we have

$$\upsilon_n = \upsilon_1/n.$$

If $n \to \infty$ then $\upsilon_n \to 0$ and the corresponding acceleration $a_n \to 0$.

A combination of eqns. (2) and (3), after some algebra, also yields

$$R_n = (\hbar^2/k_em_ee^2)n^2 \quad \ldots (3).$$

By setting n = 1 into this equation and after some algebra, we obtain

$$R_n = a_0n^2.$$

So, when $n \to \infty$ then $E_n \to 0, R_n \to \infty, \upsilon_n \to 0$ and $a_n \to 0$.

When an excited electron from the orbit with the highest energy $E_\infty = 0$ *J* returns to the ground state energy orbit it emits a photon with an energy of 13.6 eV. The radius of this highest energy orbit or the distance between an electron in this orbit and the proton is infinite. A question arises: how long it takes the electron to reach the ground state? Since $R_n \to \infty$ then the time for this transfer $t \to \infty$.

Let us assume that the energy of the highest energy electron orbit is not zero but it has an exceedingly low value, E_i. In this case, we would deal with n_i, R_i and t_i having finite but exceedingly large values. The corresponding speed υ_i and acceleration a_i would have an exceedingly low value.

When the electron reaches the ground state orbit its speed υ_1 would be about 2.2×10^6 *m sec*$^{-1}$ [1] and its average speed would be $\upsilon_{av} = (\upsilon_1 + \upsilon_i)/2 \approx \upsilon_1/2 \approx 1 \times 10^6$ *m sec*$^{-1}$.

[3] We know that $m_e\upsilon_nR$ is the angular momentum of an electron in its orbit which is, according to Bohr's model, quantized.

Premović [1] estimated that an electron situated at a distance of Bohr's radius from the proton would reach an acceleration of about 10^{30} m sec^{-2} or $a_1 \approx 10^{30}$ *m* sec^{-2}. Its average acceleration would be $a_{av} = (a_1 + a_i)/2$ or $a_{av} \approx 5 \times 10^{29}$ *m* sec^{-2}.

Since $a_{av} = \upsilon_{av}/t_i$ find that $t_i = \upsilon_{av}/ a_{av} \approx 1 \times 10^6$ *m* $sec^{-1}/5 \times 10^{29}$ *m* $sec^{-2} \approx 2 \times 10^{-24}$ *sec*. This time interval is about 2000 times smaller than the smallest time interval measured to date: 10^{-21} *sec*.

We can estimate E_i another form of the Heisenberg relations relating to energy and time. In equation form

$$E_i t_i = h$$

or

$$E_i = h/t_i.$$

Plugging the above value of t_i, we get $E_i \approx 10^{-10}$ *J*. However, now we face a problem. We know that $R_i = \upsilon_{av} \times t_i$ or $R_i \approx 1 \times 10^6$ *m* $sec^{-1} \times 2 \times 10^{-24}$ *sec* $\approx 2 \times 10^{-18}$ *m*. This is impossible since, as we noted above, R_i has to be exceedingly large.

Reference

P. I. Premović, *The Coulomb electron speed and acceleration in Bohr's model of the hydrogen atom*. The General Science Journal, April 2023.