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The Hydrogen Molecule and Deuterium Atom in the Light of de Broglie's Theory*

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General chemistry textbooks state that hydrogen molecule (H_2) consists of two protons and two electrons.[1] According to this model, protons exist in its nucleus as two independent particles.[2] However, this picture does not just agree with the picture of this molecule based on de Broglie's hypothesis of the wave nature of particles.

The de Broglie equation can be used to describe the wave nature of the proton and the H_2 nucleus in motion. Usually, this expression is expressed in the following form

$$\lambda = h/m\upsilon \quad \ldots (1)$$

where λ is the wavelength, h (= 6.63×10^{34} *J sec*)[3] is Planck's constant and m is the mass of a particle moving at a speed υ.

If H_2 travels with a non-relativistic speed υ its wavelength is then

$$\lambda(H_2) = h/m(H_2)\upsilon \quad \ldots (2)$$

where $m(H_2)$ is the rest mass of this molecule.

The wavelength of protons in this moving H_2 is

$$\lambda(H^+) = h/m(H^+)\upsilon \quad \ldots (3).$$

As the rest mass of the electron is negligible compared to the rest mass of the proton $m(H^+)$ then $m(H^+) = 1/2m(H_2)$. Plugging this term into eqn. (3) and taking into account eqn. (2) we get

$$\lambda(H^+) = 2\lambda(H_2) \quad \ldots (4).$$

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* For the sake of clarity, I have modified the first part of a previous version of this communication which refers to de Broglie's model of H_2.

[1] It is believed that in this molecule the two electrons are shared by both nuclei, so the integrity of each atom is completely lost in the molecule.

[2] The attraction of each electron by the two protons generates a force that pulls the protons toward each other and balances the repulsive force between protons and the repulsive force between electrons.

[3] To avoid confusion in further text, the SI units are given in italics.

The experimental bond (equilibrium) bond distance of H_2 is about 7.4×10^{-11} *m*. Taking into account that the proton radius is about 0.85×10^{-15} *m*, this distance is equal to about 43500 proton diameters. Thus, one concludes that H_2 consists of two separate protons having the same wavelength $\lambda(H^+)$.

Let us now assume that the proton waves of H_2 interact with each other. The superposition of these waves can result in two types of interference depending on whether their waves are in phase or out of phase: constructive and destructive interference.

In their constructive interference, the resultant wave would have twice large an amplitude as the proton but its wavelength would be the same as the proton wavelength. In the destructive interference, the two proton waves would cancel each other so there would be no resultant wave. Since they are associated with H_2 their speed is identical then, according to eqn. (4), the wavelength of H^+ is twice the wavelength of H_2.

In contrast, de Broglie's eqn. (1) implies that the nucleus of H_2 does not consist of two separate protons but of a particle whose mass is equal to the twice proton mass and the wavelength equals half of the proton wavelength [see eqn. (4)]. In other words, de Broglie's concept implies that the protons do not exist separately inside the H_2 nucleus. As far as we are aware, this claim has not been demonstrated experimentally.

Most astronomers and cosmologists believe Universe's formation started with the Bing Bang about 13.8 Gy ago. Atomic hydrogen (H) comprises about 90 % of the current Universe by number density or about 75 % of the Universe by mass. It was created in the early Universe after the Big Bang event.

In the early Universe, protons were produced abundantly. They were exposed to enough high temperatures (or having very high kinetic energy) to fuse to form the diproton nucleus. This nucleus is unstable and one proton converts into a neutron so that a deuteron (deuterium nucleus) D^+ (or $_2{}^1D$) results, releasing a positron e^+ and a neutrino ν_e. The diproton nucleus is, however, unstable and one proton converts into the neutron so that a deuteron (deuterium nucleus) D^+ results, in releasing a positron e^+ and a neutrino ν_e. In the equation form

$$p + p = D^+ + e^+ + \nu_e \quad \ldots (5).$$

This reaction is extremely slow because it is endothermic as neutrinos released carrying energies of 0.42 *MeV* or 6.7×10^{-14} *J*.

The diproton formation followed by a production of deuterium occurs also in the Sun and other similar stars. The deuterium continues in further fusion reactions fueling the Sun.

In the early Universe, neutrons were also abundantly present. It is a widely held view that in this Universe the fusion reaction of proton and neutron creates a composite stable D^+. In the equation form

$$p + n \rightarrow D^+ \quad \ldots (6).$$

The formation of D^+ is from proton and neutron would be expected to have mass

$$m_p + m_n = 3.3476 \times 10^{-27}\ kg$$

where m_p and m_n are the mass of the proton and neutron at rest. The observed mass of D^+ is 3.3436 × 10^{-27} *kg* and the mass defect for the D^+ formation process is 4 × 10^{-30} *kg* which is the equivalent of about 2.24 *MeV* or 3.6 × 10^{-13} *J*. The fusion reaction (6) is thus exothermic and thus more probable in the early Universe than the reaction (5).[4]

Therefore, the nucleus of D^+ consists of the "fused" proton and neutron containing the total mass equal to the mass of the proton and neutron combined.

Let us assume that the proton and neutron of D^+ attract each other by a force F_G described by Newton's gravitation force equation

$$F_G = G(m_p m_n)/R^2 \quad \ldots (7)$$

where F_G is the gravitational force, G is the gravitational constant (equal to 6.67 × 10^{-11} $m^3\ kg^{-1}\ sec^{-2}$) and R is the distance between their centers.[5] Let us first assume that this distance is equal approximately to the bond distance of H_2 about 7.4 × 10^{-11} *m*.

The m_p and m_n masses are approximately the same: ca. 1.7 × 10^{-27} *kg*. Thus, we can write $m_p = m_n = m = 1.7 \times 10^{-27}\ kg$ and eqn. (7) can be written as

$$F_G = Gm^2/R^2 \quad \ldots (8).$$

We know that

$$a_G = F_G/m$$

where a_G is Newton's gravitational acceleration. Combining this equation and eqn. (8) we get

$$a_G = Gm/R^2 \quad \ldots (9).$$

Plugging into this equation the above values of G, m and R (= 1.7 × 10^{-11} *m*) we obtain $a_G \approx 4 \times 10^{-16}$ $m\ sec^{-2}$. This acceleration would be much higher when the proton and neutron approach each other just before forming a deuteron [see (6)].

The strong attractive (nuclear) force of gravity F_Γ acts between a proton and neutron during their collision (or fusion) reaction generating deuteron

$$F_\Gamma = \Gamma m^2/R^2$$

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[4] Note here that all possible mechanisms in the generation of deuterium are still incomplete [1].

[5] Hypothetically, this attraction starts from infinity.

or

$$a_\Gamma = \Gamma m/R^2 \quad \dots (10)$$

where Γ is the strong (nuclear) gravitational constant Γ and R is the distance between them. However, this force is very short-range and it is only "active" at distances of about 10^{-15} *m*. It is reasonable to assume that F_Γ would be negligible at a distance of 10^{-14} *m*. At this distance, a_G would be about 10^{-9} $m\ sec^{-2}$.

The strong gravitational constant Γ was assessed for several various cases ranging from 10^{25} to 10^{32} $N\ m^2\ kg^{-2}$ [(see Wikiversity: Strong gravitational constant)]. For example, its value obtained from Fermi's weak coupling constant is equal to approximately 6.94×10^{31} $N\ m^2\ kg^{-2}$.

It is reasonable to assume that the distance between proton and neutron just before collision (or fusion) is equal approximately to the charge radius of deuteron about 2.13×10^{-15} *m*. Plugging into eqn. (9) the above values for Γ (= 6.94×10^{31} $N\ m^2\ kg^{-2}$), m (= 1.7×10^{-27} *kg*) and R (= 2.13×10^{-15} *m*) we find

$$a_\Gamma \approx 3 \times 10^{34}\ m\ sec^{-2}.$$

It means that between 10^{-14} *m* and about 2×10^{-15} *m* proton (or neutron) would be accelerated about 10^{40} times.

This acceleration would also be when the proton and neutron approach each other just before forming a deuteron [see (6)]. With this average acceleration, the proton (or neutron) would reach the speed of light c ($\approx 3 \times 10^8$ $m\ sec^{-1}$) for $\Delta t \approx 10^{-26}$ *sec*.

The quantum mechanical expression for the energy-time uncertainty is

$$\Delta E \Delta t \approx h \quad \dots (11).$$

This relates the minimum uncertainty in energy ΔE of a particle and its change during a time interval Δt. Solving for ΔE and substituting $\Delta t \approx 10^{-26}$ *sec* gives $\Delta E \approx 7 \times 10^{-8}$ *J*. This uncertainty in energy is about 2×10^5 times higher than the internal energy of deuteron of about 3.6×10^{-13} *J*. It is reasonable in this case to assume that ΔE is about this energy then the time interval Δt has to be 2×10^{-21} *sec* [eqn. (11)]. By that time the speed of a proton (or neutron) just before collision (or fusion) would be about $2 \times 10^{-21}\ sec \times 10^{34}\ m\ sec^{-2} \approx 2 \times 10^{13}\ m\ sec^{-1}$ or about 70000 times higher than the speed of light.

The strong (nuclear) acceleration can be presented as $a_\Gamma = d\upsilon/dt$ where dυ and dt represent the corresponding changes in speed and time. Special relativity limits dυ with the speed of light then $a_\Gamma < c/dt$ or

$$dt < 10^{-26}\ sec.$$

It is generally accepted that the Planck time t_p ($\approx 10^{-43}$ *sec*) is the smallest time interval that has a physical meaning. However, the smallest time interval measured to date is about 10^{-21} *sec*.[6] Therefore, dt can be from 10^{-43} *sec* to 10^{-26} *sec*. The question is now: Which value of dt is right?

Let us first assume that the proton and neutron are moving with a non-relativistic speed υ[7] just before collision (or "fusion"). After this event, the proton and neutron move together as deuteron. Their total non-relativistic kinetic energy $E_k = m\upsilon^2$. A part of this energy would be transformed into the internal energy of deuteron and a part into its kinetic energy. As we noted above, this internal energy is 3.6×10^{-13} *J*. Therefore, $E_k > 3.6 \times 10^{-13}$ *J* or after a bit of algebra and calculation we find that $\upsilon > 1.4 \times 10^7$ *m sec⁻¹*.

Consider now a relativistic case. The formula for the total relativistic kinetic energy of proton and neutron is

$$E_{rk} = 2m(\gamma - 1)c^2$$

where $\gamma = 1/\sqrt{(1 - \upsilon^2/c^2)}$ is a relativistic factor. Plugging into this equation the above values of m and c and having in mind that $E_{rk} > 3.6 \times 10^{-13}$ *J* we find that the speed of a proton (or neutron) just before a collision (or fusion) is close to the speed of light. In this case, dt is about 10^{-26} *sec*.

Quantum mechanics allows us to write a general expression for the energy-time principle:

$$\Delta E \Delta t \approx h.$$

This expression relates the minimum uncertainty in energy dE of any particle system changing during a time interval Δt. Solving for ΔE and substituting the above values for Δt gives

$$dE \approx 10^{-7}\ J - 10^{10}\ J.$$

Something is wrong but the question is: what?

Reference

[1] R. Yang, B. Chen, H. Zhao et al., *Test of conformal gravity with astrophysical observations*. arXiv: 1311.2800v1 [gr-qc] 12 Nov 2013.

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[6] Plugging into dt < c/a_Γ dt ($\approx 10^{-21}$ *sec*) and a_Γ ($\approx 3 \times 10^{34}$ $m\ sec^{-2}$), we obtain $c > 1.6 \times 10^{13}$ $m\ sec^{-1}$.

[7] We do not know the speed of a proton (or neutron) at any moment of its motion.