the general science journal

Gravitational Acceleration and the Speed of Light

Pavle I. Premović
Laboratory for Geochemistry, Cosmochemistry and Astrochemistry, University of Niš, P.O. Box 224, 18000 Niš, Serbia

Newton's gravitational law states that the gravitational force between two astronomical objects is proportional to their masses and inversely proportional to the square of the distance between their centers. We assume that these two objects are spherical and the mass of one object is M and the mass of the other object is m and that M >> m.[1]
The gravitational force is given by

$$F = GMm/R^2$$

where G (= 6.67 × 10^{-11} $N\,m^2\,kg^{-2}$)[2] is the gravitational constant and R is the distance between their centers.

The gravitational force is attractive and causes that the object of the mass m (hereinafter the m object) to deflect from a straight-line path to a circular path (orbit) around the object with the mass M (hereinafter the M object). We know in this orbit the m object is exposed to the gravitational centripetal force: $F_p = GMm/R^2$ and the centrifugal force: $F_f = m\upsilon^2/R$. These two forces are balanced: $F_p = F_f$ or $GMm/R^2 = m\upsilon^2/R$. In order the m object to move one-dimensionally along the straight line towards the M object $F_p > F_f$ or, after some algebra,

$$GM/R > \upsilon^2.$$

The mass of the M object can be presented by the following equation

$$M = (4/3)\pi R_M^3\rho \quad \ldots (1)$$

where R_M and ρ are its radius and density. Introducing this term into the above inequality, we obtain, after some algebra,

$$\upsilon^2 < 4GR_M^3\rho/R.$$

When the m object reaches the object M then $R = R_M$. Plugging into this inequality R_M instead R we get, after some algebra,

---

[1] In astronomy, we usually deal with a radius of a massive object much greater than the radius of a much smaller object (see below).
[2] To avoid confusion in further text, the SI units are given in italics.

$$\upsilon < 2R_M\sqrt{(G\rho)}.$$

The maximum gravitational force F_{max} between these two objects would be near the surface of the M object. It can be mathematically expressed as

$$F_{max} = GMm/R_M^2.$$

Newton's second law of motion states that the acceleration of an object equals the net force acting on it divided by its mass. In this case, the maximum acceleration of the m object can be represented by the equation

$$a_{max} = (F_{max}/m) = GM/R_M^2 \quad \ldots (3).$$

Combining this equation and eqn. (1) and a bit of algebra we get

$$a_{max} \approx 4GR_M\rho \quad \ldots (4).$$

Suppose now that the density of the M object is 1000 $kg\ m^{-3}$. (For comparison, the average density of the Earth is about 5500 $kg\ m^{-3}$). Plugging this value in eqn. (4) the above values of G and ρ we obtain

$$a_{max} \approx 2.7 \times 10^{-7}R_M.$$

Using this equation, we find that the m object would reach the gravitational acceleration of $c \times 1\ sec^{-1} \approx 3 \times 10^8\ m\ sec^{-2}$ if the radius of the M object is about 10^{12} *km*.[3,4] The Earth's mass is about 6×10^{24} *kg* and its radius is about 6371 *km*.[5] The Sun is the star at the center of the Solar System it has a mass of about 2×10^{30} *kg* and a radius is about 7×10^5 *km*. On average, the Sun's density is about 1400 $kg\ m^{-3}$.

The Milky Way's mass is about 3×10^{42} *kg* although the mass of a galaxy is difficult to estimate with any accuracy. It has a density less than a billionth of a billionth of the density of air on Earth then according to eqn. (4): a_{max} is almost zero.

UY Scuti, the star near the center of the Milky Way, is the largest known star in the Universe its radius is about 1.2×10^9 *km* and its density is probably about the density of the Sun. It appears that the gravitational acceleration, at least, in the Milky Way is much less than $c \times 1\ sec^{-1}$.

As we pointed out above, gravitational acceleration is described as the object receiving an acceleration due to the gravitational force F acting on it or in equation F = ma. From the eqn. (1), we find that the gravitational acceleration of the m object is equal

$$a = GM/R^2.$$

[3] Of course, c is the speed of light and is equal to $2.99792 \times 10^8\ m\ sec^{-1}$ or approximately $3 \times 10^8\ m\ sec^{-1}$.

[4] A simple calculation shows the m object would reach the acceleration of about $3 \times 10^9\ m\ sec^{-2}$ if R_M of the M object is about 10^{13} km.

[5] Just to remind you, near Earth's surface, the gravity acceleration is approximately 9.81 $m\ sec^{-2}$.

As we have shown above, this acceleration is limited by the speed of light c and it must be $a < c \times sec^{-1}$. It can be mathematically expressed as

$$GM/R^2 < c \times sec^{-1}$$

or

$$M/R^2 < c \times sec^{-1}/G.$$

Substituting the above values of $c \times sec^{-1}$ ($\approx 3 \times 10^8\ m\ sec^{-2}$) and G ($= 6.67 \times 10^{-11}\ N\ m^2\ kg^{-2}$) into this inequality, we obtain

$$M/R^2 < 4.5 \times 10^{18} \text{ (expressed in } kg\ m^{-2}\text{)}.$$

After a bit of algebra, we have the following expression

$$R < 4.7 \times 10^{-10}\sqrt{M} \quad \ldots (5)$$

Venus has a nearly circular orbit with an eccentricity of 0.007. Its mass is $4.87 \times 10^{24}\ kg$ and it is much smaller than mass of the Sun equal to $1.989 \times 10^{30}\ kg$. Using eqn. (5) we find that $R < 670\ km$ which is much smaller than the radius of the Sun is about $7 \times 10^5\ km$.

Currently, it appears the Segue 2 in the Aries constellation is the one closest to the Earth orbiting the Milky Way. It is a dwarf galaxy that has an approximately round shape with a half-light radius of about $1.1 \times 10^{11}\ km$. Its mass is about $2.2 \times 10^{36}\ kg$. Using eqn. (5) we find that $R < 8 \times 10^8\ km$ which is much smaller than the radius of the Milky Way about $5 \times 10^{17}\ km$.