

The Minimum Speed of a Free Non-Relativistic Massive Particle and Its Uncertainty in Position: a Brief Note

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The simplest form of Heisenberg's uncertainty principle is:

$$\Delta p \Delta x \geq h \quad \dots (1)$$

where Δp is the uncertainty in momentum of a particle and Δx is its uncertainty in position both in the same direction, and h ($= 6.63 \times 10^{-34}$ J sec) is Planck's constant. In words, it is impossible simultaneously to know the momentum and position of a particle to infinite accuracy.

Once the uncertainty in momentum Δp is found, the uncertainty in speed Δv can be found from:

$$\Delta p = m \Delta v$$

and vice versa. Of course, this equation is valid for a free non-relativistic massive particle (FNMP) with a mass at rest m_0 :

$$\Delta p = m_0 \Delta v \quad \dots (2).$$

Premović [1] has derived the following equation for the minimum speed of this particle within the non-relativistic framework:

$$v_{\min} = h/m_0 D_0 \quad \dots (3).^1$$

In this derivation, he followed the analogy with light² and assumed (rather reasonably) that this particle is spherical and has a diameter at rest, D_0 . Assuming the minimum speed must be at least equal to its uncertainty then Δv for the FNMP must be equal or larger than v_{\min} . So, eqn. (2) has the following form:

$$\Delta p \geq m_0 v_{\min}.^3$$

¹ This speed is a constant characteristic of a given FNMP [1].

² From analogy with the standing wave in a spring, it is easy to find that that the minimum speed $v_{\min} = h/2m_0 D_0$ and the minimum uncertainty in position $\Delta x = 2D_0$. The difference between the initial assumptions from "analogy of light" and from "analogy of the standing wave in a spring", the reader can find in footnote 2 of the reference [1].

³ This is a constant characteristic of a given FNMP.

The minimum uncertainty in momentum for a given FNMP is then:

$$\Delta p_{\min} = m_0 v_{\min}.$$

Combining this equation and equation (3) and after a bit of algebra, we have:

$$\Delta p_{\min} = h/D_0.^4$$

In words, the minimum uncertainty in momentum of a (reasonably spherical) free non-relativistic massive particle is equal to Planck's constant divided by its diameter. This is in direct contradiction with Heisenberg's uncertainty principle which allows for an FNMP an infinitely small uncertainty in momentum [2, and references therein].

For an FNMP eqn. (1) can be written as follows:

$$m_0 v_{\min} \Delta x \geq h.$$

The minimum of this product is

$$m_0 v_{\min} \Delta x = h.$$

This speed is constant and characteristic of a given FNMP [1].

Combining this equation and equation (3) and after again a bit of calculus, we get the maximum in position of an FNMP:

$$\Delta x = D_0.^5$$

In words, the maximum uncertainty in position of a (reasonably spherical) non-relativistic free massive particle is equal to the diameter of this particle. This is also in direct contradiction with Heisenberg's uncertainty principle which allows for this particle an infinitely large uncertainty in position [2, and references therein].

Since eqn. (3) is also valid for a free relativistic massive particle [1] this conclusion can be applied to this particle. However, as far as this author is aware the incompatibility between Special relativity and the Uncertainty principle is still a problem of modern physics that requires a sound solution.

References

[1] Pavle I. Premović, *The Minimum Speed of a Free Massive Particle*. The General Science Journal, September 2021.

[2] V. Todorinov, P. Bosso, S. Das, *Relativistic Generalized Uncertainty Principle*. Annals Phys. 405, 92-100 (2019).

⁴ This is a constant characteristic of a given FNMP.

⁵ This is a constant characteristic for FNMP in question.