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The Newton Law of Gravitation: Another Look

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In 1687, Newton put forward the Universal law of gravitation to explain the observed motions of planets and their moons. This law states that every object in the Universe attracts another object with a force along a line joining them. For the sake of simplicity, here we will deal with a planet (like Uranus) with a circular orbit around a star (like the Sun).

The force is directly proportional to the product of the masses of the astronomical objects and inversely proportional to the square of the distance between them.[1]

This law can be expressed in the equation

$$F_G = G(mM)/R^2 \quad \ldots (1)$$

where F_G is the gravitational force, G is the gravitational constant (equal to 6.67×10^{-11} *N m kg*$^{-2}$), m is the mass of the planet, M is the mass of the star and R is the distance between their centers. M is much greater than m or M >> m. The planet is exposed to the gravitational centrifugal force: $F_f = m\upsilon^2/R$ and the centripetal force: $F_p = GMm/R^2$. These two forces are balanced: $F_f = F_p$ or

$$m\upsilon^2/R = GMm/R^2$$

After some algebra we get

$$\upsilon^2 R = GM \quad \ldots (2)$$

Since the mass of star M is constant then GM is constant. Introducing into eqn. (2) the given values for G and M, we find $R\upsilon^2$ (= GM) = 1.34×10^{20} *N m kg*$^{-1}$.

If $R \to \infty$ then $\upsilon \to 0$. However, the product of infinitely large and infinitely small quantities cannot be finite. Therefore, R cannot be infinitely large but extremely large but finite; υ cannot be infinitely small[2] but extremely small but finite.

Multiplying both sides of eqn, (2) with 1/2 and after some arrangement we get

$$1/2m\upsilon^2 = ½(GM/R).$$

[1] The derivations and discussion are similar to those in [1].
[2] Since υ is squared, we are dealing with an "infinitesimally infinite" small quantity.

The left side of this equation is the kinetic energy of the planet, $E_{K,}$ and the right one is its potential energy E_p. In other words, $E_K = 1/2E_p$. Following the above discussion on the limits of R and υ, we reason that E_k and E_p cannot be infinitely small, but extremely small but finite.

The total energy E of the planet is the sum of its kinetic energy, $1/2m\upsilon^2$ and its potential energy, GM/R, or $E = ½(m\upsilon^2) + GM/R = 3/2(GM/R)$. If $R \rightarrow \infty$ then $E \rightarrow 0$. However, as E_p (= GM/R) is only extremely small and finite, E can only be numerically extremely small but finite.