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Bohr's Model of the Hydrogen Atom: The Total Energy of the Electron

Pavle I. Premović
Laboratory for Geochemistry, Cosmochemistry and Astrochemistry,
University of Niš, pavleipremovic@yahoo.com, Niš, Serbia

In an early model of the hydrogen atom (the Bohr model), the centripetal force acting on an electron in a circular orbit around the proton is $m\upsilon^2/R$, where m ($\approx 9.1 \times 10^{-31}$ kg) is the electron's mass, υ is its velocity, and R is the orbit's radius. From Coulomb's Law, the force of attraction between the electron and the proton is e^2/R^2, e ($\approx 1.9 \times 10^{-19}$ C) is the elementary charge on the proton and the electron. To maintain a stable orbit, the electron's centripetal force and its Coulomb force to the proton must be equal; E_p after ignoring their signs, we write

$$m\upsilon^2/R = k_e e^2/R^2 \quad \ldots (1)$$

where k_e is the Coulomb constant (= $8{,}988 \times 10^9$ N m^2 C^{-2}).

After some algebra, this equation can be written as

$$R\upsilon^2 = k_e(e^2/m) \quad \ldots (2).$$

Experimental evidence suggests that the electron of the hydrogen atom orbits the proton with a non-relativistic speed with respect to an Earth's observer. If so, then $k_e(e^2/m)$ is a constant for this observer.

Introducing into eqn. (2) the given values for k_e, e^2 and m, $R\upsilon^2$ [= $k_e(e^2/m)$] = 260 m^3 sec^{-2}.
If $\upsilon \to 0$ then $R \to \infty$. However, the product of infinitely small[1] and infinitely large quantities cannot be finite. Therefore, for the Earth's observer, υ cannot be infinitely small but extremely small but finite; R cannot be infinitely large but extremely large but finite.

However, the theory of Special relativity limits the speed of electron to less than the speed of light c ($\approx 3 \times 10^8$ m sec^{-1}). It is reasonable to suggest that the upper non-relativistic limit of the electron υ is about half of the speed of light c or $\upsilon \approx 0.5c$ {see also [1]}. At this speed, its classical kinetic energy is about equal to its relativistic one. If $\upsilon \approx 0.5c$ then, applying eqn, (2), we estimate that the electron is in the orbit with the lowest possible radius $R_{min} \approx 3 \times 10^{-13}$ m.

[1] Since υ is squared, we are dealing with an "infinitesimally infinite" small quantity.

To explain Bohr's model of the hydrogen atom, in which the electron is only allowed to orbit at certain distances from the nucleus, de Broglie postulated that there is a wave associated with the electron with the wavelength $\lambda = h/p$ where $p = mv$ is the electron momentum and h (= 6.63×10^{-34} J sec) is Planck's constant. Only waves that fit exactly around the orbit are allowed.

The Bohr-de Broglie model of the hydrogen atom predicted the speed of electron present in the first orbit of the hydrogen atom to be approximately 2.2×10^{6} m sec^{-1} (≈ 0.06c) and the radius of this orbit a_0 to be about 5.3×10^{-11} m. Introducing $\upsilon = 0.06c$ into eqn. (2), R is about 5.4×10^{-11} m.

Multiplying both sides of eqn, (2) with 1/2 and after some arrangement we get

$$1/2m\upsilon^2 = 1/2k_e(e^2/R).$$

The left side of this equation is the kinetic energy of the electron in a hydrogen atom, E_K, and the right one is its potential energy E_p. In other words, $E_K = -1/2E_p$. Following the above discussion related to the limits of R and υ, we reason that E_k and E_p cannot be infinitely small but extremely small but finite.

The total energy E of the electron on the hydrogen atom is the sum of its kinetic energy, $1/2m\upsilon^2$ and its potential energy, $-k_ee^2/R$, or $E = ½(m\upsilon^2) - k_ee^2/R = 1/2k_ee^2/R$. If $R \rightarrow \infty$ then $\upsilon \rightarrow 0$, so $E \rightarrow 0$. However, as E_p (= $-k_ee^2/R$) is only extremely small and finite, E can be numerically extremely small but finite and negative.

Elementary physics states that the allowed electronic energy states of the hydrogen atom are given by

$$E = E_0/n^2$$

where E_0 is the ground state electronic energy of this atom and its value is − 13.6 eV[2] and n is the positive integer called the energy quantum number. So, if $n \rightarrow \infty$ then $R \rightarrow \infty$ and $E \rightarrow 0$. Since E can be numerically extremely small but finite and negative, n can be extremely large but finite.

[2] Since $E \rightarrow 0$ is arbitrarily chosen for $R \rightarrow \infty$, then E_0 appears to be negative.