the general science journal

New Mathematical Ways to Derive the Uncertainty Expressions Between Energy/Position and Momentum/Time

Pavle I. Premović
Laboratory for Geochemistry, Cosmochemistry and Astrochemistry,
University of Niš, pavleipremovic@yahoo.com, Niš, Serbia

Heisenberg's uncertainty principle states that it is impossible to determine accurately both the momentum and the position of a moving microscopic particle simultaneously. The basic relation of this principle is given by

$$\Delta p \Delta x \geq h \quad \ldots (1)$$[1]

where Δp is the uncertainty in momentum, Δx is the uncertainty in position and $\hbar$ is the Planck constant ($= 6.63 \times 10^{-34}$ *J sec*).

Another form of Heisenberg's principle relates uncertainty in energy ΔE to uncertainty in time Δt of the above microscopic particle[2]

$$\Delta E \Delta t \geq h \quad \ldots (2).$$

In our papers [1, 2], using a relatively simple mathematical procedure, we derived two new relations of the Heisenberg uncertainty principle for a non-relativistic particle

$$\Delta E \Delta x < \hbar c \text{ (the energy-position uncertainty)} \quad \ldots (3) \text{ and}$$

$$\Delta p \Delta t > \hbar/c \text{ (the momentum-time uncertainty)} \quad \ldots (4)$$

where c ($\approx 3 \times 10^8$ *m sec*$^{-1}$) is the speed of light.

Heisenberg's microscope is a thought experiment proposed by Heisenberg that provides an argument for the uncertainty principle of non-relativistic quantum mechanics for the momentum and position. In this experiment to determine the electron's position, he designed a microscope using gamma rays instead of visible light, as in a classic optical microscope.

Our thought experiment is similar to Heisenberg's microscope experiment. A gamma ray photon (hereinafter photon) collides with a moving electron, as shown in Fig. 1. The incident photon has the energy $E = h\nu$, where ν is its frequency. After the collision, the particle has a speed υ. For the sake of simplicity, we will start our derivations using *h*.

---

[1] The absolute minimum uncertainty of $\hbar$ or $\hbar/2$ is far more common than the value *h*.

[2] It appears that the meaning of uncertainty in time is not yet clarified.

To derive eqns. (3) and (4) we have two options. The first is based on the assumption that the uncertainty in the position of the particle Δx is approximately equal to the wavelength of the photon λ. That is,

$$\Delta x \approx \lambda.$$

The uncertainty in the energy of the particle ΔE has to be much smaller than $h\nu$ or

$$\Delta E << h\nu$$

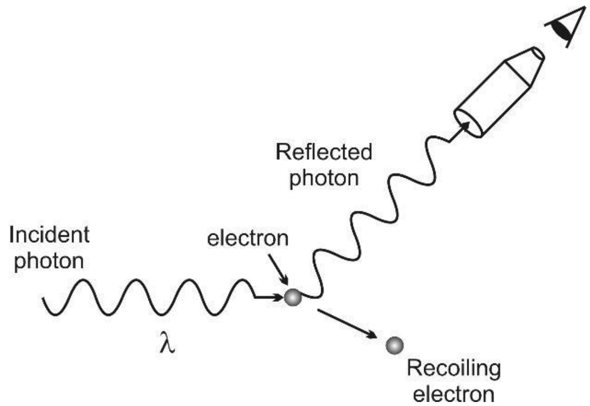

Fig. 1. Our presentation of the Heisenberg microscope thought experiment.[3]

Elementary physics states that $\lambda = c/\nu$. Introducing this term into the left side of this inequality and after a bit of algebra, we get

$$\lambda \Delta E << hc$$

Taking into account the above assumption we have

$$\Delta E \Delta x << hc \text{ or } \Delta E \Delta x < \hbar c$$

Dividing the relation (1) by c, we have

$$\Delta p \Delta x/c \geq h/c.$$

Introducing λ (= c/ν) instead of Δx into this equation

$$\Delta p/\nu \geq h/c.$$

$\Delta E \Delta t \geq h$ then $E\Delta t >> h$. Substituting E with $h\nu$ and after some algebra, we find that $1/\nu << \Delta t$.[4]

Hence,

$$\Delta p \Delta t >> h/c \text{ or } \Delta p \Delta t > \hbar/c.$$

[3] This figure is based on an original illustration by Harshit Rautela.
[4] Gamma rays have frequencies above 3×10^{19} Hz and the duration of the particle energy change $\Delta t >> 1/\nu$.

The second option is based on the following equation

$$\Delta E = \upsilon \Delta p \quad \ldots (4)$$

derived and explained by Jain [3]. Combining this equation and $\Delta E \Delta t \geq h$, we get

$$\upsilon \Delta p \Delta t \geq h$$

or

$$\Delta p \Delta t \geq h/\upsilon.$$

As $c >> \upsilon$, then

$$\Delta p \Delta t >> h/c \text{ or } \Delta p \Delta t > \hbar/c.$$

Multiplying the relation (1) by υ, we have

$$\upsilon \Delta p \Delta x = \Delta E \Delta x \geq h\upsilon.$$

As $hc >> h\upsilon$,

$$\Delta E \Delta x < hc \text{ or } \Delta E \Delta x < \hbar c.$$

Finally, some textbooks and other educational materials state that the uncertainty in position Δx is approximately equal to the de Broglie wavelength of the particle. That is,

$$\Delta x \approx \lambda_B = h/p.$$

Combining this approximation with the relation (1) and after a bit of algebra, we find that $\Delta p \Delta x$ is approximately equal to or larger than the momentum p. Of course, this is impossible. Therefore, the assumption that $\Delta x \approx \lambda_B$ leads to a contradiction $\Delta p \approx p$.

References

[1] P. I. Premović, *The Energy-position and the momentum-time uncertainty expressions*. The General Science Journal.
[2] P. I. Premović, *Some further notes on the energy-position/momentum-time uncertainty expressions for a non-relativistic particle*. The General Science Journal.
[3] M. C. Jain, *Quantum mechanics: A textbook for undergraduates* (second edition). PHI Learning Private Limited, Delhi (2017).