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One-Dimensional Perfectly Elastic Collision of a Microscopic Particle with a Wall: a Quantum Leap in the Speed Direction

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We will first consider a one-dimensional perfectly elastic collision[1] of a non-relativistic massive particle[2] with a wall. Fig. 1. From elementary physics, we know that the speed of this particle υ before (moving along the positive x-axis) and after (moving along the negative x-axis) this collision, will be constant but its direction will be reversed while the wall remains stationary. (At the instant when the particle reverses its direction, its speed is zero). In other words, the particle will bounce off the wall moving with the speed υ along the negative x-axis.

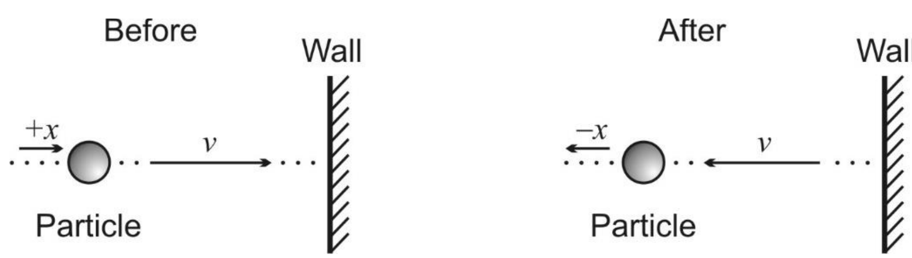

Figure 1. A one-dimensional elastic collision of a massive particle with a wall.

According to elementary physics, at the start of its reversal the massive particle speed υ must be zero [1]. In that case, the particle will be accelerated from 0 to υ during the time interval dt

$$a = \upsilon/dt \quad \ldots (1).$$

Special relativity limits this acceleration with the speed of light c (= 2.99792×10^8 m sec^{-1}) and it must be < c/1 sec. Introducing this inequality into eqn. (1) and after a bit of algebra we get

[1] A one-dimensional collision occurs when there is only motion along one axis..
[2] Of note here is that perfectly elastic collisions can happen only with subatomic particles. Elastic collisions can be achieved only with particles like microscopic particles like electrons, protons or neutrons.

$$dt > (\upsilon/c) \times 1 \text{ sec} = (\upsilon_n/c_n) \times 1 \text{ sec}$$

or, in general,

$$dt > (\upsilon/c) \times 1 \text{ unit of time} = (\upsilon_n/c_n) \times 1 \text{ unit of time} \quad \dots (2)$$

where υ_n and c_n are the numerical values of the non-relativistic particle speed υ and speed of light c. Since the speed of light c is constant the time interval dt for the particle to reach the speed υ after the collision is proportional to the unit of time used.

If we use the second as a unit of time then the particle will reach speed υ for dt > second. On the other hand, if we use, for example, as the unit of time the year or century than dt > year or > century. The only solution of this issue is dt = 0. This can be defined as a quantum leap in the speed direction. It also implies three corollaries: (1) the motion direction of the particle is changed without changing its speed or at the start of its direction reversal the particle speed is not zero but υ as before the collision, (2) the reversal of this direction is instantaneous, (3) the particle will travel no distance in the reverse direction before reaching the speed υ.

In the case of massive relativistic particle, we write

$$dt/\sqrt{(1- \upsilon^2/c^2)} > (\upsilon/c) \times 1 \text{ sec}/\sqrt{(1- \upsilon^2/c^2)}$$

After a bit of algebra, we obtain a result identical to the inequality (2) without making any approximation. The above three corollaries are also valid now.

We know that the wavelength of matter waves (de-Broglie waves) associated with a moving particle is $\lambda = h/mv$. For a particle at rest $\upsilon = 0$ means $\lambda = \infty$. This means that the de Broglie waves are generated only when the particle is in motion. Hence, the interpretation of a particle with a wavepacket is possible only with the particle in motion. Since in the above collision, the particle speed is not zero, at the moment when the particle changes direction, it can be interpreted as a wavepacket before and after the collision.

The above consideration can be applied od the other cases of one-dimensional (head-on elastic) collisions of two particles with of equal and unequal masses as between the two particles with equal or unequal masses moving in the same direction.

Reference

[1] L. Kirkpatrick, G. E. Francis, *Physics: A Conceptual World View*. Brooks/Cole (Belmont, USA), 2010.