

## The Ultimate Speed without Infinite Mass

Musa D. Abdullahi, U.M.Y. University  
P.M.B. 2218, Katsina, Katsina State, Nigeria  
E-mail: [musadab@msn.com](mailto:musadab@msn.com), Tel: +2348034080399

### 1 Introduction

Failure to accelerate electrons, the lightest particles known in nature, to a speed beyond that of light might be what misled physics, early in the 20<sup>th</sup> century, into the wilderness of special relativity. The theory of special relativity makes the mass  $m$  of a moving particle increase with its speed  $v$  according to the formula:

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (1)$$

where  $m_o$  is the rest mass and  $c$  the speed of light in a vacuum.. At speed  $c$  mass  $m$  becomes infinitely large. Since an infinite mass cannot be accelerated any faster by any force, equation (1) appears plausible. It answered the question why a particle cannot be accelerated beyond the speed of light. However, the issue of infinite mass remained an intractable problem.

### 2 Speed of Propagation of Electric Force

A better explanation for the speed of light being an ultimate limit is obtained by considering the fact that an electric force is propagated at the velocity of light  $c$  and that the velocity of transmission of the force, relative to an electron moving with velocity  $v$ , is  $(c - v)$ . The electron can be accelerated to the speed of light  $c$  and no faster. The accelerating force  $F$  on an electron of charge  $-e$  and mass  $m$  moving at time  $t$  with velocity  $v$  at an angle  $\theta$  to  $F$ , in an electrostatic field of intensity  $E$  and magnitude  $E$ , is proposed as given by the equation:

$$\mathbf{F} = \frac{eE}{c}(\mathbf{c} - \mathbf{v}) = \frac{-e\mathbf{E}}{c} \sqrt{c^2 + v^2 - 2cv\{\cos(\theta - \alpha)\}} = m \frac{d\mathbf{v}}{dt} \quad (2)$$

where  $\alpha$  is the small angle of aberration,  $(\theta - \alpha)$  is the angle between  $v$  and  $c$  such that

$$\sin\alpha = \frac{v}{c} \sin\theta \quad (3)$$

The relationship between the vectors  $E$ ,  $F$ ,  $v$  and  $c$  and the angles  $\theta$  and  $\alpha$  is depicted in Figure 1. For  $\theta = 0$  or  $\pi$  radians, there is rectilinear motion with emission of radiation. For  $\theta = \pi/2$  radians there is circular revolution with constant speed and no radiation of energy.

### 3 Equations of Rectilinear Motions

For an accelerated electron where  $\theta = 0$ , equations (2) and (3) give the accelerating force  $F$ , in rectilinear motion, as:

$$\mathbf{F} = -eE \left(1 - \frac{v}{c}\right) \hat{\mathbf{u}} = -m \frac{dv}{dt} \hat{\mathbf{u}} \quad (4)$$

where  $\hat{\mathbf{u}}$  is a unit vector in the direction of the electric field  $E$ .

The solution of equation (4) for an electron accelerated from zero initial speed, is

$$\frac{v}{c} = 1 - \exp\left(-\frac{at}{c}\right) \quad (6)$$

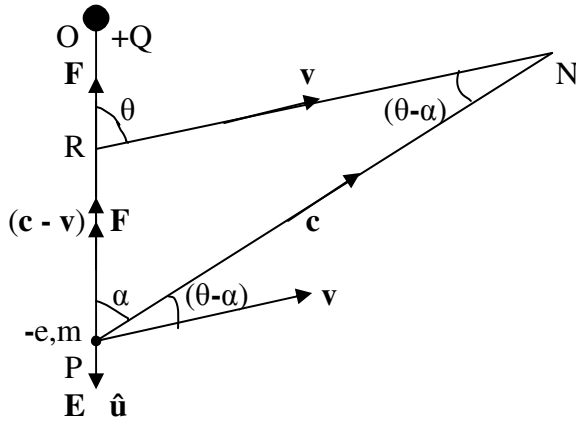


Figure 1 Vector diagram depicts angle of aberration  $\alpha$  as a result of an electron of charge  $-e$  and mass  $m$  moving, at a point  $P$ , with velocity  $v$ , at an angle  $\theta$  to the accelerating force  $F$ . The unit vector  $\hat{u}$  is in the direction of the electrostatic field of intensity  $E$  due to a stationary source charge  $+Q$  at the origin  $O$ .

where  $a = eE/m = eE/m_o$ , is a constant. The electron will be accelerated to maximum speed  $c$ .

For a decelerated electron where  $\theta = \pi$  radians, equations (2) and (3) give  $F$  as:

$$\mathbf{F} = -eE \left( 1 + \frac{v}{c} \right) \hat{u} = m \frac{dv}{dt} \hat{u} \quad (7)$$

Solving equation (7) for an electron decelerated from the speed of light  $c$ , gives:

$$\frac{v}{c} = 2 \exp \left( -\frac{at}{c} \right) - 1 \quad (8)$$

The electron is brought to a stop and then accelerated to reach an ultimate speed  $v = c$ .

#### 4 Equations of circular motion

For  $\theta = \pi/2$  radians, motion is in a circle of radius  $r$  with constant speed  $v$  and acceleration  $(-v^2/r)\hat{u}$ . Equations (2) and (3), with mass  $m = m_o$  and noting that  $\cos(\pi/2 - \alpha) = \sin \alpha = v/c$ ,

$$\mathbf{F} = -eE \sqrt{1 - \frac{v^2}{c^2}} \hat{u} = -m_o \frac{v^2}{r} \hat{u} \quad (9)$$

$$eE = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \frac{v^2}{r} = \zeta \frac{v^2}{r}$$

$$\zeta = \frac{eEr}{v^2} = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (10)$$

#### 5 Conclusion

Equation (1) for “ $m$ ” and equation (10) for  $\zeta$  are identical but obtained from two different points of view. In equation (1), the quantity “ $m$ ” increases with speed  $v$ , becoming infinitely large at speed  $c$ . In equation (10), mass  $m$  remains constant at the rest mass  $m_o$ , and  $\zeta$  is the ratio of magnitude of electrostatic force ( $eE$ ) on a stationary electron to the centripetal acceleration ( $v^2/r$ ) in circular motion. This quantity  $\zeta$  may become infinitely large at the speed of light  $c$ , without any difficulty. So equation (1) is correct only for circular motion of an electron but not from the point of view of mass increasing with speed but on the basis of accelerating force reducing with speed in accordance with equation (2), Thus we have the ultimate speed  $c$  without infinite mass to bring great relief to physicists all over the world.