the general science journal

The Reduced Compton Wavelength and the Energy-Position/Momentum-Time Uncertainties

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The Compton scattering is an elastic collision of the x-ray or gamma photons with free electrons (or loosely bound valence shell electrons). This scattering is demonstrated in Fig. 1. In this communication, we combine Heisenberg's uncertainty principle and Compton scattering. Note, that some of the expressions (including their derivations) of this communication can be found in modern physics textbooks.

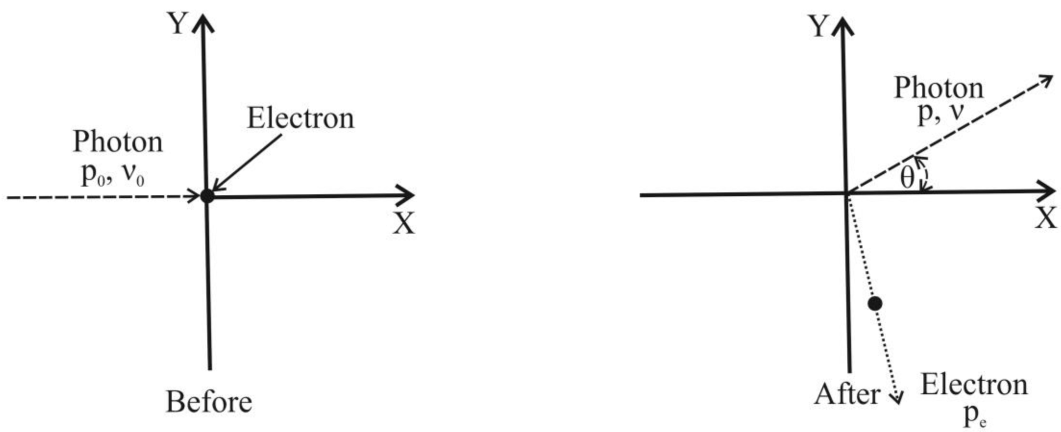

Figure 1. Schematic representation of Compton scattering, in which an incoming photon scatters from an electron at rest.

The electron of the rest mass m_e has the rest energy of $E_r = m_ec^2$. The Compton scattered electron (hereinafter the Compton electron) possesses relativistic kinetic energy, the relativistic speed υ_e and the linear momentum $p_e = m_e\upsilon_e$.

Before scattering, let the photon move along the positive x-axis having an energy $E_0 = h\nu_0$, frequency ν_0, wavelength λ_0 and linear momentum $p_0 = h\nu_0/c$, where h (= 6.63 × 10^{-34} J sec) is the Planck constant and c (≈ 3×10^8 m sec^{-1}) is the speed of light. After scattering, the photon moves at an angle θ off the x-axis with an energy $E = h\nu$, frequency ν, wavelength λ and momentum $p = h\nu/c$.

The conservation of linear momentum of the electron after the collision yields,

$$p_e^2 = p_0^2 + p^2 - 2p_0p\cos\theta.$$

The conservation of energy gives,

$$(p_e c)^2 = (h\nu_0 - h\nu)^2 + 2m_e c^2(h\nu_0 - h\nu).$$

Combining these two expressions leads to the standard Compton formula

$$\lambda - \lambda_0 = (h/m_e c)(1 - \cos\theta).$$

Here $h/m_e c$ is called the standard Compton wavelength of the electron, λ_c, and it has a value of 2.43 10^{-12} m. This formula supposes that the scattering occurs in the rest frame of the electron. The energy $h\nu_0$ of the incident photon with this wavelength is equal to the rest mass energy $m_e c^2$ of the electron. In the following discussion, we will take the reduced Compton wavelength $\lambda_{rc} = \hbar/m_e c$ (= 3.87×10^{-13} m), where $\hbar$ (= $h/2\pi$ =1.05×10^{-34} J $\sec^{-1}$) is the reduced Planck constant.

If the Heisenberg uncertainty principle holds for the Compton electron[1] then

$$\Delta p_e \Delta x_e \geq \hbar.$$

This inequality can be expressed as:

$$\Delta p_e \Delta x_e = \alpha\hbar \quad \text{... (1)}$$

where $\alpha \geq 1$. In this case, quantum mechanics predicts that the uncertainty in the position of the Compton electron Δx_e must be greater than the reduced Compton wavelength $\hbar/m_e c$ or $\Delta x_e > \lambda_{rc} = \hbar/m_e c$.[2] Accordingly, we can write

$$\Delta x_e = \alpha\hbar/m_e c \quad \text{... (2)}$$

where $\alpha > 1$.

If the uncertainty in energy Δp_e of the Compton electron is higher than $m_e c$ then the uncertainty in its energy ΔE_e ($> m_e c^2$) would be high enough to create an electron-positron pair. Suppose that

$$\Delta p_e = b m_e c \quad \text{... (3),}$$

where $b \leq 1$. Thus, for the Compton electron the momentum-position expression can be written as

$$\Delta p_e \Delta x_e = b\alpha\hbar \quad \text{... (4)}$$

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[1] This is a crucial assumption. In general, it is worth mentioning that the lower limit of $\hbar/2$ for $\Delta p \Delta x$ is rarely attained; more usually $\hbar$ even h.

[2] Since the standard Compton wavelength λ_c (= $h/m_e c$) and the de Broglie wavelength ($\lambda_{DB} = h/m_e v$) are both greater than λ_{rc} (= $\hbar/m_e c$). So, if $\Delta x_e > \lambda_{DB}$ or $> \lambda_c$ then $\Delta x_e > \lambda_{rc}$.

with $b\alpha = a$, see equation (1).

By differentiating the relativistic equation for the total energy E_e of the Compton electron $E_e^2 = (p_e c)^2 + (m_e c^2)^2$, we obtain

$$\Delta E_e = \upsilon_e \Delta p_e \quad ... (5).$$

If we multiply both sides of equation (5) with Δx_e and use equation (4) we get

$$\Delta E_e \Delta x_e = \upsilon_e \Delta p_e \Delta x_e = b\alpha\hbar\upsilon_e \quad ... (6).$$

According to Special relativity, the speed of the Compton electron υ_e cannot be greater than c. It is reasonable to assume that $1 \le a \le 2$[3] and that υ_e is relativistic when it is equal to or larger than 0.5c. In this case $b\alpha\upsilon_e < 2c$. So equation (6) can be rewritten

$$\Delta E_e \Delta x_e < 2\hbar c \; ... (7).$$

Recently, employing the uncertainty relation for momentum-position, we have derived the same energy-position uncertainty expression for a relativistic particle with the speed $\upsilon \ge 0.5c$ [1]. This derivation was based on the same two assumptions as above for the Compton electron, and these are (A): the speed of light c is independent of the motion of the observer, as postulated by Special relativity and (B): $1 \le a \le 2$.

We also derived the expression for the momentum-time position for a relativistic particle with the speed $\upsilon \ge 0.5\, c$ [1]:

$$\Delta p_p \Delta t > \hbar/c \quad ... (8).$$

The energy time-uncertainty relation for the Compton electron is

$$\Delta E_e \Delta t \ge \hbar \quad ... (9).$$

As we derived above $\Delta E_e = \upsilon_e \Delta p_e$. So, relation (9) can be rewritten

$$\upsilon_e \Delta p_e \Delta t > \hbar \quad ... (10).$$

Since the speed of the Compton electron υ_e after interaction with the photon cannot be greater than c. In this case (10) we arrive at

$$\Delta p_e \Delta t > \hbar/c.$$

Thus, inequality (8) holds for the Compton electron.

As we pointed out in footnote 1 the value of $\hbar/2$ is seldom attained; the values $\hbar$ and h are more common. Thus, the energy-position expression (7) can be written as $\Delta E_e \Delta x_e < \hbar c$ or $\Delta E_e \Delta x_e <$

[3] It is reasonable to expect that $1 < \alpha \le 2$.

$2hc$. Similarly, the momentum-time expression (8) can be reformulated as $\Delta p_e \Delta t > \hbar/2c$ or $\Delta p_e \Delta t > h/c$.

Reference

[1] P. I. Premović, *The Energy-position and the momentum-time uncertainty expressions.* The General Science Journal, December 2021.