

Gravitational Acceleration Constant

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The gravitational force between two astronomical objects having masses m and M is given by Newton's law of gravity, which says that the attractive force between them is proportional to the product of their masses and inversely proportional to the square of the distance between their centers. This law can be expressed in the equation

$$F_G = GmM/R^2 \quad \dots (1)$$

where F_G is the gravitational force, G is the gravitational constant (equal to

$6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ sec}^{-2}$)¹. For the sake of simplicity, we will assume that the mass of the more massive astronomical object M (hereinafter the M object) is much greater than the mass of the less massive astronomical object (hereinafter the m object) or $M \gg m$.

According to Newton's second law of motion, the acceleration of an object (a) equals the net force (F) acting on it divided by its mass (m). This can be represented by the equation

$$a = F/m \quad \dots (1).$$

In this case, we have

$$F_G = ma_G \quad \dots (2)$$

where a_G is the gravitational acceleration of the m object received due to the gravitational force F_G acting on it. From the eqn. (2), we find that the gravitational acceleration is equal

$$a_G = GM/R^2.$$

Special relativity limits the acceleration of the m object to the speed of light c ($\approx 3 \times 10^8 \text{ m sec}^{-1}$) and it must be $a_G \ll c/1 \text{ sec}$ [1].

Indeed, suppose $a_G = c/1 \text{ sec}$. In expression,

$$a_G = c \times \text{sec}^{-1} (\approx 3 \times 10^8 \text{ m sec}^{-2})$$

or

$$GM/Rc^2 = c \times \text{sec}^{-1}.$$

This is the minimum possible distance between the m and M objects or $R > R_c$.

Hence,

$$M/R_c^2 = c \times \text{sec}^{-1}/G.$$

Substituting known values of $c \times \text{sec}^{-1}$ and G into this inequality, we obtain

$$M/R_c^2 = 4.5 \times 10^{18} \text{ (expressed in } kg \text{ m}^{-2}\text{)} \dots (3).$$

$M/R_c^2 = c \times \text{sec}^{-1}/G = 4.5 \times 10^{18} \text{ kg m}^{-2}$ represents its maximum value obtained assuming that the gravitational acceleration $a_G = c \times 1 \text{ sec}^{-1}$. We name it as the gravitational acceleration constant $\gamma = 4.5 \times 10^{18} \text{ kg m}^{-2} > M/R^2$.

Using eqn. (1) and after a bit of algebra, we have the following expression for the distance R_c

$$R_c = 4.7 \times 10^{-10} \sqrt{M} \dots (4).$$

Example: In the Solar system, Venus has a nearly circular orbit with an eccentricity of 0.007. Its mass is about $4.87 \times 10^{24} \text{ kg}$, and it is significantly smaller than the mass of the Sun. Introducing the mass of the Sun of about $1.989 \times 10^{30} \text{ kg}$ into eqn. (4), we find that $R_c = 670 \text{ km}$. In contrast, the radius of the Sun is much larger, about $7 \times 10^5 \text{ km}$.

The Milky Way's mass is about $6 \times 10^{42} \text{ kg}$, although the mass of a galaxy is difficult to estimate with any accuracy. Currently, it appears the Segue 2 in the Aries constellation is the one closest to the Earth orbiting the Milky Way. It is a dwarf galaxy with a mass of about $2.2 \times 10^{36} \text{ kg}$. Using eqn. (4), we estimate that R_c would be about $1.15 \times 10^{12} \text{ km}$. In contrast, the Milky Way's radius is much larger, about $4.75 \times 10^{17} \text{ km}$.

Reference

[1] P. I. Premović, *Acceleration of the non-relativistic uniformly accelerated astronomical objects and the speed of light*. The General Science Journal.