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Gravitational Acceleration and the Speed of Light

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Newton's gravitational law states that the gravitational force between two astronomical objects is proportional to their masses and inversely proportional to the square of the distance between their centers. We assume that these two objects are spherical and the mass of one object is M and the mass of the other object is m and that M >> m.[1]
The gravitational force is given by

$$F = GMm/R^2$$

where G (= 6.67 × 10^{-11} *N* m^2 kg^{-2})[2] is the gravitational constant and R is the distance between their centers.

The gravitational force is attractive and causes the object of the mass m (hereinafter the m object) to deflect from a straight-line path to a circular path (orbit) around the object with the mass M (hereinafter the M object). We know in this orbit the m object is exposed to the gravitational centripetal force: $F_p = GMm/R^2$ and the centrifugal force: $F_f = m\upsilon^2/R$. These two forces are balanced: $F_p = F_f$ or $GMm/R^2 = m\upsilon^2/R$. For the m object to move one-dimensionally along the straight line towards the M object $F_p > F_f$ or, after some algebra,

$$GM/R > \upsilon^2.$$

The mass of the M object can be presented by the following equation

$$M = (4/3)\pi R_M^3\rho \quad \ldots (1)$$

where R_M and ρ are its radius and density. Introducing this term into the above inequality, we obtain, after some algebra,

$$\upsilon^2 < 4GR_M^3\rho/R.$$

When the m object reaches the object M then $R = R_M$. Plugging into this inequality R_M instead of R we get, after some algebra,

$$\upsilon < 2R_M\sqrt{(G\rho)}.$$

[1] In astronomy, we usually deal with the radius of an object with a mass M that is much greater than the radius of a much smaller object with a mass m (see below).
[2] To avoid confusion in further text, the SI units are given in italics.

The maximum gravitational force F_{max} between these two objects would be near the surface of the M object. It can be mathematically expressed as

$$F_{max} = GMm/R_M^2.$$

Newton's second law of motion states that the acceleration of an object equals the net force acting on it divided by its mass. In this case, the maximum acceleration of the m object can be represented by the equation

$$a_{max} = (F_{max}/m) = GM/R_M^2 \quad \ldots (3).$$

Combining this equation and eqn. (1) and a bit of algebra we get

$$a_{max} \approx 4GR_M\rho \quad \ldots (4).$$

Suppose now that the density of the M object is 1400 $kg\ m^{-3}$. (On average, the Sun's density is about 1400 $kg\ m^{-3}$). Plugging this value in eqn. (4) and the values of G and ρ we obtain

$$a_{max} \approx 4 \times 10^{-7} R_M.$$

Using this equation, we find that the m object would reach the gravitational acceleration $a \geq c \times 1\ sec^{-1} \approx 3 \times 10^8\ m\ sec^{-2}$ if the radius of the M object is $R_M \geq 7.5 \times 10^{14}\ km$.[3, 4] For example, the Sun is the star at the center of the Solar System it has a mass of about $2 \times 10^{30}\ kg$ and a radius is about $7 \times 10^5\ km$. No object in this (or in a similar planetary system in the Milky Way galaxy) can have an acceleration $a \geq c \times 1\ sec^{-1} \approx 3 \times 10^8\ m\ sec^{-2}$.

[3] Of course, c is the speed of light and is equal to approximately $3 \times 10^8\ m\ sec^{-1}$.

[4] A simple calculation shows the m object would reach the acceleration of about $3 \times 10^9\ m\ sec^{-2}$ if the R_M of the M object is about 10^{13} km.