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The Dimensionality of the Geiger-Nuttall Law: a simple but interesting note

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Abstract. It appears that the natural logarithm form of the empirical Geiger-Nuttall law is not dimensionally correct. It is suggested a new type of this form as well as four new non-exponential forms of this law.

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Derivations and Discussion. One of the milestones of quantum mechanics was the formulation of the Geiger-Nuttall law [1, 2]. This law states that there is a simple empirical relation between the α-decay energy Q_α and the half-life of heavy alpha (α)-emitter $t_{1/2}$

$$\log t_{1/2} = A + B/\sqrt{Q_\alpha}$$

where A and B are constants. Let us convert this equation into a natural logarithm (LN) form

$$\ln t_{1/2} = A + B/\sqrt{Q_\alpha}$$

where a = 2.303A and b = 2.303B.[1] The kinetic energy, E_α, of the emitted α-particle is slightly less (about 0.4 % for heavy α-emitters, such as the uranium α-emitters) than Q_α. Therefore, the last equation can be written as follows

$$\ln t_{1/2} = a + b/\sqrt{E_\alpha} \quad \ldots (1).$$

After the conversion of this equation into the exponential form we have

$$t_{1/2} = e^{a + b/\sqrt{E_\alpha}} \quad \ldots (2).$$

The dimension[2] of the left side of this expression is T but its right side is dimensionless. Hence, eqn. (2) is dimensionally incorrect. The simplest way to make this equation dimensionally correct is to multiply its right side with the constant θ equal or close to 1 and expressed in time unit of $t_{1/2}$ (e. g. seconds)

$$t_{1/2} = e^{a + b/\sqrt{E_\alpha}} \times \theta.$$

[1] $\ln t_{1/2} = 2.303\log t_{1/2}$

[2] Just to remind the reader that M, L and T are the symbols for basic dimensions: mass, length and time, respectively.

Since lnθ is equal or close to 0 the LN form of this equation is identical to the initial eqn. (1) of the Geiger-Nuttall law.

There is another way to resolve the above dimensionality issue. Instead, the eqn. (1) derived from empirical Geiger-Nuttall law we may propose the following equation

$$t_{1/2} = ab/\sqrt{E_\alpha} \quad \ldots (4).$$

The LN form of this equation is $\ln t_{1/2} = \ln a + \ln(b/\sqrt{E_\alpha})$ where a and b are the constants. The last equation is a linear form of $t_{1/2}$ vs. $1/\sqrt{E_\alpha}$. After some rearrangement of eqn. (4) we get

$$t_{1/2} = a\sqrt{(b^2/E_\alpha)}.$$

Keeping in mind that the dimension of E_α[3] is ML^2T^{-2}, a dimensional analysis shows that this equation is dimensionally correct for the fourth following cases: 1) if the dimension of a is L then the dimension of b^2 must be M; 2) if the dimension of a is T then the dimension of b^2 must be ML^2T^{-2}, i. e. b^2/E_α must be dimensionless; 3) if the dimension of a is M then the dimension of b^2 must be $M^{-1}L^2$, and; finally 4) if a is dimensionless then the dimension of b^2 must be ML^2.

The detailed analysis of each of the above cases, including eqns. (1) and (2), will be the subject of the author's further study.

Recently, Premović [3] derived a new mathematical form for the Geiger-Nuttall law. This is expressed by the following equation

$$t_{1/2} = u \times w \times 1/\sqrt{E_\alpha}$$

where u and w are the constants. According to Premović [7], the dimension of u is L (analogous to the above case 1) and the dimension of $(w \times 1/\sqrt{E_\alpha})$ is $[L^{-1} \times T]$ or the dimension of w is M Therefore, the left side of this expression is dimensionally consistent with its right side.

References

[1] H. Geiger and J.M. Nuttall, *The ranges of the α particles from various radioactive substances and a relation between range and period of transformation.* Phil. Mag., 22, 613-621 (1911).
[2] H. Geiger and J.M. Nuttall, *The ranges of α particles from uranium,* Phil. Mag., 23, 439-445 (1912).
[3] P. I. Premović, *The lifetimes of ^{238}U nuclei, the escaping attempts of their alpha particles and the Geiger-Nuttall law.* The General Science Journal, December 2021.

[3] Since $E_\alpha = 1/2mv^2$ (where m is the mass of the α-particle and v is its speed) and the dimension of v is LT^{-1} then the dimension of E_α is ML^2T^{-2}