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The Concept of "Mass" in Extended Classical ElectroDynamics (ECED)

on the example of Ideal Particle (IP)

Yuri Keilman
AltSci@basicisp.net

The Extended CED (see my article, Classical ElectroDynamics, or Phys Essays, **11,** p.34 (1998)) is based upon Maxwell's equations:

$$div\vec{H} = 0; curl\vec{E} + \frac{1}{c}\dot{\vec{H}} = 0; div\vec{E} = \frac{4\pi}{c}J^0; curl\dot{\vec{H}} - \frac{1}{c}\dot{\vec{E}} = \frac{4\pi}{c}\vec{J} \quad \textbf{(1)}$$

and the Dynamics Equation:

$$J^0\left(\frac{k_0^2 c}{4\pi}\vec{E} + \nabla J^0 + \frac{1}{c}\dot{\vec{J}}\right) + \vec{J}\times\left(\frac{k_0^2 c}{4\pi}\vec{H} - curl\,\vec{J}\right) = 0 \quad \textbf{(2)}$$

which replaces the Relativistic Dynamics Equation of conventional CED. This Dynamics Equation is obtained as a result of **the requirement of conservation** (notice that conservation is not a consequence but the requirement which produces a dynamics equation) of the energy-momentum tensor:

$$T_i^k = -\frac{1}{4\pi}F_{ia}F^{ka} + \frac{1}{16\pi}F_{ab}F^{ab}\delta_i^k - \frac{4\pi}{k_0^2c^2}j_ij^k + \frac{2\pi}{k_0^2c^2}j_aj^a\delta_i^k \quad \textbf{(3)}$$

In another turn, the energy-momentum tensor (3) is the result of a minimum action with the Lagrangian:

$$\Lambda = -\frac{1}{4\pi}F_{ab}F^{ab} - \frac{2\pi}{k_0^2c^2}J_aJ^a \quad \textbf{(4)}$$

with respect to **arbitrary variation of a metric tensor in a flat (zero curvature) space**.

The Dynamics Equation (2) satisfied in the vacuum (J^k=0) and should be satisfied inside the elementary particle. On the boundary between these regions (which is a "free boundary" and a characteristic surface of the Equation (2)) there should be:

$$\mu^2 = (J^0)^2 - (J^1)^2 - (J^2)^2 - (J^3)^2 = 0 \quad \textbf{(5)}$$

This requirement comes from equation (2). From (1) and (2) we can find that inside the elementary particle (but not on the boundary itself) should be satisfied the equation:

$$\Box J^k + k_0^2 J^k = 0 \quad \textbf{(6)}$$

Definition: **Material continuum** exists inside elementary particles and can not be divided into a system of material points. Newton's mechanics disappears without a trace inside material continuum: no velocity, no acceleration, no force. **Kinematical** state of material continuum is defined by the field of current density J^k. **Dynamics** is defined by the equation (2). We deny casual connection between any two points on the line of current J^k so that these lines are not what is usually called "world lines of material points". The consequence of this is that vector J^k can be space-like as well as time-like. In the expression: $J^k = \mu u^k$, u^k is not a 4-velocity but just a unit vector because there is no material point -- no casual connection.

STATIC SOLUTIONS: Ideal Particle (IP+, IP)

One of the ways of finding solutions of the system (1,2) is to take a solution of (6) in the inside region of the particle, find the electric and magnetic field inside by the formulas:

$$\frac{ck_0^2}{4\pi}\vec{E} = -\nabla J^0 - \frac{1}{c}\dot{\vec{J}}; \quad \frac{ck_0^2}{4\pi}\vec{H} = curl\,\vec{J} \quad \textbf{(7)}$$

Then, having current J^k, choose suitable boundary of the particle so that to satisfy the condition (5) on a free boundary. After that we can find corresponding electric and magnetic fields in vacuum watching for continuity of $\vec{E}, \vec{H}$.

If there is no dependence of time then equation (6) turns to a vector Helmholz equation. The systematic description of the solutions of this equation can be found in the book by Morse and Feshbah "Mathematical Methods of Theoretical Physics".

The simplest static solution of the system (1, 2) in free space (we call it Ideal Particle IP, or IP+) is:

$$J^0 = c\alpha R_0(z),\ 0 \le z \le z_1; \quad J^0 = 0,\ z_1 \le z < \infty; \quad \vec{J} = 0 \quad \textbf{(8)}$$

where $z = k_o r$, $R_o(z)=\sin(z)/z$ - the spherical Bessel function, α is an arbitrary constant. It is clear that to satisfy the condition (4) we have to take $z_1=n\pi$. Only in this case our solution will be truly Ideal Particle (IP). But we keep z_1 arbitrary calling the solution IP+. IP+ satisfies the system everywhere except the boundary of the particle. The full charge can be found by integration:

$$e(z_1) = 4\pi\int_0^{r_1} J^0 r^2 dr = \frac{4\pi\alpha c}{k_0^3} z_1^2 R_1(z_1) \quad \textbf{(9)}$$

where: $R_1(z)=\sin(z)/z^2 - \cos(z)/z$ - another spherical Bessel function. The electric field of IP is:

$$\frac{k_0}{4\pi}E_r = \alpha R_1(z),\ 0 \le z \le z_1; \quad \frac{k_0}{4\pi}E_r = \alpha R_1(z_1) z_1^2 / z^2,\ z_1 \le z < \infty; \quad \vec{H} = 0 \quad \textbf{(10)}$$

To find the mass we should integrate T^{00} (taken from (3)) over the volume:

$$T^{00}=\frac{1}{8\pi}(E^2+H^2)-\frac{2\pi}{k_0^2c^2}[(J^0)^2+(\vec{J})^2]$$

$$c^2m=\int T^{00}r^2\sin(\theta)drd\theta d\varphi=\frac{8\pi^2}{k_0^2c^2}\int[\frac{k_0^2c^2}{16\pi^2}E^2-(J^0)^2]r^2dr=$$

$$=\frac{8\pi^2\alpha^2}{k_0^5}\{\int_0^{z_1}[R_1^2(z)-R_0^2(z)]z^2dz+R_1^2(z_1)z_1^4\int_{z_1}^{\infty}\frac{dz}{z^2}\}= \qquad \textbf{(11)}$$

$$=\frac{8\pi^2\alpha^2}{k_0^5}\{-z_1^2R_0(z_1)R_1(z_1)+z_1^3R_1^2(z_1)\}=-\frac{8\pi^2\alpha^2}{k_0^5}z_1^2R_1(z_1)\cos(z_1)$$

At the first glance the result of the integration looks like positive and negative mass is possible. But a plotting reveals that at z_1 more than ~1.6 the mass stays positive with only a very little exceptions.

Another important integral in this theory is:

$$c^2m_1\equiv\int T^{11}dV=-\frac{8\pi^2\alpha^2}{3k_0^5}z_1^3R_0^2(z_1);\quad m_1=m_2=m_3 \qquad \textbf{(12)}$$

This integral stays negative at any z_1 but it is zero for IP (at $z_1=n\pi$).

If IP+ moves with a constant velocity V in x^1 direction then we can find any data about the solution just by a Lorentz transformation. We can perform the corresponding integration and find energy and linear momentum. They appear to be:

$$P^0=\gamma(m+m_1V^2);\quad P^1=\gamma(m+m_1)V;\quad \gamma=(1-V^2)^{-1/2} \qquad \textbf{(13)}$$

We have a "rest mass" which is m, and we have a "linear momentum mass" which is $P^1/(\gamma V)=m+m_1$. The last one was discussed by R. Feynman in his "Lectures" on p.28-3. He called it "electromagnetic mass". But "linear momentum mass" looks more appropriate. These masses the same for IP and they are different for IP+. The reader may say why bother with IP+ at all? The reason is: our models of the real particles -- electron, proton, neutron,... are based not on IP but on IP+.

In order to decide which mass is more important we put IP+ in an external constant electric field. We expressed the field that is produced by the particle through the **retarded** potentials and substituted this field in the dynamics equation (2). After a rather lengthy derivation we managed to obtain a solution around the initial moment of time t=0 in the third order (the initial velocity assumed to be zero). This solution confirmed Newton's dynamics equation including radiation reaction. The "acceleration mass" appear to coincide with the rest mass of IP+. That means that the rest mass is more important than the "linear momentum mass".

There was also a test of stability of IP. The result is: IP is an unstable equilibrium. Nothing is known about the stability of IP+.

Also I'd like to remind here that "illegal models" (illegal because they do not satisfy the system on the particle boundaries -- like IP+) of electron and proton (see Phys. Essays, **11**, p.34) give external radius ~1.2 Fermi for the electron, and ~1.1 Fermi for the proton/neutron. These numbers are in agreement with Hofshtadtter's experiments, if these experiments are interpreted in real (not reciprocal) space.