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Symmetry in Classical Physics

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1. Introduction

This article is an attempt to revise and clarify the meaning of the term "symmetry", at least with respect to **classical physics** and Einstein's **Special Relativity** theory.

Let us begin with the article of David J. Gross (the Professor at Princeton University): "Symmetry in Physics: Wigner's Legacy", Physics Today, Dec. 1995, p. 46.

In the introduction part of his article, which concerns symmetry principles in classical theoretical physics, D. Gross states:

"Until the 20th century, principles of symmetry played little role in theoretical physics. **Conservation laws**, especially those of energy and momentum, were considered to be of fundamental importance. But these were regarded as consequences of the **dynamical laws of nature**, rather than as consequences of the **symmetries that underlay these laws**. Maxwell's equations, formulated in 1865, embodied both Lorentz invariance and gauge invariance. But these symmetries of electrodynamics were not fully appreciated for 40 years or more." (Bold mine). Directly further D. Gross states:

"Einstein's great advance in 1905 was to **put symmetry first**, to regard the symmetry principle as the primary feature of nature that constrains the allowable dynamical laws. Thus the **transformation properties** of the electromagnetic field were not to be derived from Maxwell's equations, as Hendrik Lorentz did, but rather were consequences of **relativistic invariance**, and indeed largely dictate the form of Maxwell's equations. This is a profound change of attitude. Lorentz must have felt that Einstein cheated."

At the end of this introductory part regarding the negative attitude towards theory of groups D. Gross states: "That this attitude has changed so dramatically over the last 60 years, so today principles of symmetry are regarded as the most fundamental part of our description of nature...".

There is more to the expression "symmetries that underlay the conservation laws". In the book by H. Goldstein "Classical Mechanics", 1980, p. 588, we read:

"A recurring theme throughout has been that symmetry properties of the Lagrangian (or Hamiltonian) imply the existence of conserved quantities. Thus, if the Lagrangian does not contain explicitly a particular coordinate of displacement, then the corresponding canonical momentum is conserved. The absence of explicit dependence on the coordinate means the Lagrangian is unaffected by a transformation that alters the value of that coordinate; it is said to be invariant, or symmetric under the given transformation. Similarly, invariance of the Lagrangian under time displacement implies conservation of energy. The formal description of the connection between invariance or symmetry properties and conserved quantities is contained in Noether's theorem."

This thread was developed by many others, for example, Wu-Ki Tung in his book "Group Theory in Physics" on p.9 in the section "Symmetry Groups of Physics" states:

"(a) Translations in Space, $\mathbf{x} \rightarrow \mathbf{x}+\mathbf{a}$, where $\mathbf{a}$ is a constant 3-vector: This symmetry, applicable to all isolated systems, is based on the assumption of *homogeneity of space,* i.e. every region of space is equivalent to every other, or physical phenomena must be reproducible from one location to another. The conservation of linear momentum is a well known consequence of this symmetry."

According to the logic of D. Gross "the symmetries", that come first, underlay dynamical laws of nature and the laws of energy and momentum conservation. What kind of symmetries is he talking about? According to H. Goldstein the one of them is invariance of the Lagrangian under 4-translations. Is it really the cause of energy-momentum conservation?

Let us consider inhomogeneous space. Suppose in some fixed point of space we have a charge that produces a constant field that depends on radius and some charged particle moves in that field. According to the dynamical laws of nature the local conservation of linear momentum still exists (by this I mean that divergence of the energy-momentum tensor is still equal to zero everywhere in space except the point of a fixed charge). The translation symmetry disappeared -- the conservation of linear momentum remains in place. This is the clear indication that the linear momentum conservation is not a consequence of the symmetry. Let us investigate the problem deeper.

2. One-Point and Two-Points Transformations of Coordinates

The concept "**transformation of coordinates**" in general is a very important one in mathematics and theoretical physics. At first it seems, that it is the law that allows us to get 4 real numbers x'^k if another 4 real numbers x^k are given. The theory of groups takes over and studies the different identities of these transformation laws. But this is not all to the concept "transformation of coordinates". The theory of groups studies a "transformation engine" only. It is also important how we use the obtained transformed real numbers x'^k. If we are **changing the coordinate system**, then x^k are the coordinates of a point P in the original coordinate system K, while x'^k are the new coordinates of the **very same** point in a new coordinate system K'. This use of the transformations we call:

One-point transformation of coordinates. We consider an arbitrary (Jacobian should be different from zero or infinity) transformation engine with the same number of original and transformed coordinates. In each point P we consider scalars (given by one real number), vectors (given by 4 ordered real numbers), second rank tensors (given by 16 ordered real numbers), and so on. The transformation laws for the covariant and contravariant components of a vector (and a tensor) are different:

$$x'^k = x'^k(x^i);\ x^k = x^k(x'^i);\ A'^k = \frac{\partial x'^k}{\partial x^i} A^i;\ A'_k = \frac{\partial x^i}{\partial x'^k} A_i;\ T'_i{}^k = \frac{\partial x^m}{\partial x'^i} \frac{\partial x'^k}{\partial x^n} T_m{}^n \quad (1)$$

If we consider the space of 4-dimensional independent variable x^k as a metric space then a metric tensor g_{ik} is considered to be given in original coordinate system K. The components of a metric tensor undergo transformation as a second rank tensor. The transformation of coordinates changes the coordinate system K, but does not change the point in space, nor does it change the scalars, vectors, and tensors that are connected to this point of space. A scalar does not change its value that is expressed by a real number. The components of vectors and tensors change, but they describe the same vectors and tensors only in a new coordinate system. All these scalars, components of vectors and tensors originally are functions of old coordinates. The arguments of all these functions (which are old coordinates) should be **expressed** (not replaced) through the new coordinates. Although point P is "every point of space", all the described above algebraic operations are performed at the very same point of space before going to another point of space, where the whole process repeats. The whole operation physically means that we are not changing a physical object, which was described originally in the coordinate system K, -- we are changing the coordinate system and giving a new description to the very same physical object. The main property of vectors (tensors) is the fact that they carry the specific identity which survives arbitrary transformation of coordinate system (meaning arbitrary "transformation engine") while their components change. To underline this fact we can call scalars, vectors, and tensors by the name **mathematical objects.** The calling of "carriers of a mathematical identity" by the name "mathematical objects" is another meaning of the term which, according to the dictionary of mathematical terms already has different meaning (group, vector space,...). But it looks that we have to live with multiple meanings of the words. To the category of "mathematical objects" we can also add points, and any multitudes of points (like lines, triangles, and so forth. These last we call **extended mathematical objects**). The coordinates of the points, the specifics of definitions of multitudes of points, the components of vectors and tensors we can call **mathematical accessory.** There is a good rule in theoretical physics: physical objects can be described only by mathematical objects; mathematical accessory has only indirect relation to the physical objects.

Two-Point Transformations of Coordinates (symmetry transformation). We can consider another scenario of using the transformed coordinates: mapping. The coordinate system remains the same, the metric tensor remains the same, but the transformed real numbers are the coordinates of another point P1 in the same space. This notion is used in the symmetry investigations of the **extended mathematical (physical) objects**. Actually, we are making a second copy of description of the original physical object in the same coordinate system. We take the physical properties (that are described by scalars, vectors and tensors) from point P1 and transfer them to point P. Along with that we keep the physical properties that were originally in point P safe. Since point P is "every" point of space, point P1 also will go over the whole space. Many questions arise as to how this transfer of data should be done. The scalars, probably, can be transferred without problems, but vectors and tensors definitely require a special treatment. To account for that we consider a "change of the form" of the vector and tensor components. Usually one prescribes the same transformations of the vector and tensor components as in one-point transformation (only without expressing the old arguments of functions through the new ones). But this should be justified because the other ways are also possible. The transformation of coordinates itself also should be "physically" justified.

It should be clear that a two-point transformation is specific to a physical object (it is not arbitrary) and can be called "symmetry transformation".

3. Incorrect use of E. Noether's Theorem

The Noether's theorem is a statement regarding two-point transformations of coordinates only. It allows us to obtain a specific conserving quantity if the Lagrangian of the physical system invariant (remains the same for both physical copies in every point of space) with respect to the specific "symmetry" transformation (two-point transformation of coordinates with specific rules as how to transfer the physical properties from one point to another). Noether's theorem can not be applied to one-point transformation of coordinate system because the Lagrangian is a true scalar (a pseudoscalar Lagrangian can be rewritten as a true scalar with the help of the square root from minus determinant of the metric tensor) and, consequently, it is invariant with respect to "arbitrary" one-point transformation. No specific consequences can be drawn from that invariance.

The misuse of Noether's theorem is its application to one-point transformations. The Theory of Groups studies the "transformation engine" only, not paying attention how this engine is applied. If we take 4-translations as the "engine", then in two-point transformation context the majority of physical Lagrangians won't be invariant. In one-point transformation context all Lagrangians will be invariant. The misuse starts with an incorrect claim that the Noether's theorem can be used for 4-translation regarding all Lagrangians that do not explicitly depend on the coordinates. This misuse continues on by choosing (without any justification) the "change of the form" of the field functions so that the outcome of the Noether's theorem gives the usual energy-momentum tensor [see H. Goldstein p.592 formulas (12-153) and (12-154), (12-155). From (12-155) it follows that on the requirement (12-153) the "change of the form" of the field function will be: $\bar{\delta}\eta_{\rho} = -\varepsilon_{\gamma}\eta_{\rho,\gamma}$. I do not see how this "change of the form" can be justified].

Contrary to D. Gross the specific dynamical laws of nature are the consequences of the requirement of minimum action with a specific Lagrangian. The conservation of energy and momentum is a consequence of the dynamical laws. There exists a unique procedure that allows us to obtain the energy-momentum tensor from the Lagrangian. In the light of this, the previous relation can be reversed: the conservation of energy and momentum is a requirement (as the minimum of action is) and the dynamical laws are the consequences.

4. "Symmetry" by the name "Relativistic Invariance"

In his article D. Gross raises the question about a more general type of "symmetry" than just 4-translation. Originally, Maxwell's equations were written in a 3-d form. They are really "embody" the possibility to be rewritten in a 4-d form without any change to their meaning. This rewriting only possible if one introduces a 4-d Lorentz metric tensor (or uses imaginary time as Minkovski did). By the term "symmetry" in this connection D. Gross means "relativistic invariance" which, in turn, means invariance under Lorentz transformation, which, in another turn, requires 4-d description (defines the "transformation properties") of the electromagnetic field. In D. Gross's terminology the term "symmetries" comes first.

I do not see why we can not reverse the chain: let us put first the "Requirement of 4-d Description". This requirement is mathematical in nature and has a clear meaning. Before we go to "symmetries" we have to elaborate on a clearer meaning of the term. From my point of view, symmetries underlay physical objects, not physical laws. The "Requirement of 4-d Description" by itself is a very fundamental change that affects all aspects of theoretical physics. It assumes that the chapter of mathematics traditionally called "4-d Geometry" is readily available. It is a mathematical requirement that applies now to any rigorous physical formula. In a way, it stands "before" any physical formula. This requirement is not a physical requirement and can not be thought of as a kind of symmetry. One can write in 4-d form as many different equations as in 3-d form. The form of Maxwell's equations obviously is not "dictated" by the "Requirement of 4-d Description".

The "Requirement of 4-d Description" was prompted by Maxwell's equations. The Lorentz transformations follow from the Lorentz metric tensor as the Lorentz metric tensor follows from the Maxwell's equations. The Theory of Special Relativity follows.

Einstein himself took the first "fruits" of the "Requirement of 4-d Description": he upgraded the Newton's Dynamics Equation to the Relativistic Dynamics Equation. This move was rich of physical consequences. But, for some reason, Einstein did not want to admit that these advances were prompted by the Maxwell's equations. Instead he put forward his famous 2 postulates as the foundation of Special Relativity. From above it should be clear that both his postulates are the consequences of the chain: Maxwell's Equations -- Requirement of 4-d Description -- Lorentz metric tensor -- Lorentz Transformations. The name "Special Relativity" stands far from the mathematical "Requirement of 4-d Description" and adheres to the postulate of equality of all "inertial observers" through which it acquires a "physical aureole".

These are among the roots why many physicists do not understand SR. Up to now people are trying to "test Lorentz Invariance" experimentally not realizing that mathematics does not need to be tested. So far, there is no mathematical alternative to the 4-d description, because 3-d description with absolute time is inconsistent from a pure mathematical standpoint. What they can find at best is improved Maxwell's equations which still have to be written in "Lorentz Invariant" form (4-d form).

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