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Derivation of Special Relativity from Maxwell's Equations

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1. This derivation relies on a full knowlege of mathematics. You must know the meaning of scalar, vector, or tensor in **n** dimensions (usually called Riemann Geometry, which actually is a Theory of **n** -dimensional supernumbers).

2. The idea of derivation is: We have 3-dimensional geometry that describes stationary objects very well. It uses 3-dimensional supernumbers (3-vectors, 3-tensors,...) to describe them. If we want to get a good physical theory that describes moving physical objects we have to employ 4-dimensional supernumbers. Maxwell's equations are written in 3-vectors with time looking pretty different from other coordinates. But may be it is just a way of writing it? If we'll be able to write it in 4-dimensional form then we'll actually introduce 4-dimensional supernumbers (and this is basically what SR is about).

3. In Maxwell's equations we have 4 independent variables: **t, x, y, z**. We can not treat **t** any different from **x, y, z**. We can not say, for example, that **t** is a universal parameter while **x, y, z** are coordinates because there is no such thing in mathematics like a vector or tensor with a parameter. Let us introduce the special notations: **x0=ct, x1=x, x2=y, x3=z**.

4. Let us take Maxwell's equations in vacuum:

$$\mathrm{div}H=0;\ \mathrm{curl}E+H_{,0}=0;\ \mathrm{div}E=0;\ \mathrm{curl}H-E_{,0}=0;$$

Here **E** and **H** indicate 3-dimensional vectors and the coma indicates a partial derivative. So that $_{,0}$ indicates partial derivative with respect to **x0**. Let us write these equations in explicit form:

$$H_{1,1}+H_{2,2}+H_{3,3}=0;\ E_{3,2}-E_{2,3}+H_{1,0}=0;\ E_{1,3}-E_{3,1}+H_{2,0}=0;\ E_{2,1}-E_{1,2}+H_{3,0}=0;$$ Equation #1

$$E_{1,1}+E_{2,2}+E_{3,3}=0;\ H_{3,2}-H_{2,3}-E_{1,0}=0;\ H_{1,3}-H_{3,1}-E_{2,0}=0;\ H_{2,1}-H_{1,2}-E_{3,0}=0;$$ Equation #2

Notice, that 3-dimensional vectors in Euclidian metric have their covariant components equal to contravariant components: $E_k=E^k$

5. Let us define a 4-dimensional second rank tensor F_{ik} in its covariant components:

$$F_{01}=E_1;\ F_{02}=E_2;\ F_{03}=E_3;\ F_{12}=-H_3;\ F_{13}=H_2;\ F_{23}=-H_1;$$

Having this we can calculate the contravariant components of a dual tensor $F^{ik}=-1/2[\mathrm{sqrt}(-g)]e^{iklm}F;$

Where **g** is the determinant of a 4-dimensional metric tensor which is yet unknown; e^{iklm} is a 4-dimensional fully antisymmetric symbol. We can assume that **sqrt(-g)=1**. We have:

$$F^{*01}=H_1;\ F^{*02}=H_2;\ F^{*03}=H_3;\ F^{*12}=-E_3;\ F^{*13}=E_2;\ F^{*23}=-E_1;$$

and these are contravariant components by calculation.

6. Let us make some suggestions about the unknown 4-dimensional metric tensor $\mathbf{g_{ik}}$**.** We suppose that the 4-dimensional coordinate system is orthogonal so that $\mathbf{g_{ik}=0}$ if **I does not =k**. Also we suppose that $\mathbf{g_{ik}}$ components are not dependent on coordinates. That makes all Kristoffel's symbols equal to zero and the covariant derivative will coincide with the partial derivative.

7. Let us calculate $\mathbf{F^{*ik}{}_{,k}}$ which will be a contravariant vector:

$$\mathbf{H_{1,1}+H_{2,2}+H_{3,3};\ -H_{1,0}-E_{3,2}+E_{2,3};\ -H_{2,0}+E_{3,1}-E_{1,3};\ -H_{3,0}-E_{2,1}+E_{1,2};}$$

We see, that if we request: $\mathbf{F^{*ik}{}_{,k}=0}$, we will get the Maxwell's equations of (1).

8. Let us calculate $\mathbf{g^{ki}F^{*ik}{}_{,k}}$. Here $\mathbf{g^{kl}}$ are contravariant components of a metric tensor so that the whole expression is a covariant vector (since $\mathbf{F_{ik}}$is covariant). We have:

$$\mathbf{g^{11}E_{1,1}+g^{2,2}E_{2,2}+g^{33}E_{3,3};\ -g^{00}E_{1,0}-g^{22}H_{3,2}+g^{33}H_{2,3};}$$

$$\mathbf{-g^{00}E_{2,0}+g^{11}H_{3,1}-g^{33}H_{1,3};\ -g^{00}E_{3,0}-g^{11}H_{2,1}+g^{22}H_{1,2};}$$

Now an important moment: we can request $g^{kl}F_{ik,l}=0$ and we will get Maxwell's equation (2) only if:

$\mathbf{g^{00}=d,\ g^{11}=g^{22}=g^{33}=-d;}$ where **d** is some constant.

These are the contravariant components of a metric tensor. The covariant components will be:

$\mathbf{g_{00}=1/d,\ g_{11}=g_{22}=g_{33}=-1/d}$;
If we want the determinant of the metric tensor to be **-1** then: **d=± 1**.

We got the metric tensor from Maxwell's equations and this is the main thing in Special Relativity. We can get Lorentz transformations just by requesting that the components of this tensor be invariant with respect to the transformation. Of course, we have to keep in mind that physical objects can be described only by scalars, vectors, tensors,… which remain the same in any coordinates (the components of a vector transform but they describe the very same vector in different coordinates); this is a general requirement.

9. The Siblings Paradox for example. Suppose we have 3 events (in coordinates x0, x1): **A(0,0); B(ct/2,Vt/2); C (ct,0)**. We have two clocks. The world line of the clock1 is **AC**. The world line of the clock2 is **ABC**. The clock2 at first makes the trip **AB**. The reading of the clock is a physical characteristic - it can be described only by a scalar - proper time, which in this case is: **SAB=1/2ct.sqrt(1-V^2/c^2)**; (we used the metric tensor). The same way we'll get: **SABC=ct.sqrt(1-V^2/c^2); SAC=ct.**

10. What actually did we get? 4-dimensional Geometry? No! Geometry can be only 3-dimensional because geometry is a physical theory that describes stationary physical objects. Three coordinates are enough to describe them. We got more a comprehensive physical theory than geometry: physical reality with moving objects requires 4-dimensional supernumbers (4-vectors, 4-tensors,…) to get a proper description.