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In-Depth Development of Classical Electrodynamics (CED)

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Abstract: There is a hope that properly developed CED will be able to play a role in unified field theory explaining electromagnetism, quantum phenomena and gravitation all together. There is plenty of work that has to be done in this way. In this article we are trying to move forward in that direction.

Key words: classical theory of field, variation principal of classical electrodynamics, generalization of the conservation requirements on energy-momentum tensor, extended classical particles, disruption surfaces, additional conditions on electromagnetic potential.

Statement of contribution: In the present article, we continue to refine the basic principles of the improved CED. Major attention is paid to the reinterpretation of E-M potential. We use these basic principles to obtain solutions that explain the interactions between constant electromagnetic field and a thin layer of a material continuum; between a constant electromagnetic field and a spherical formation of a material continuum (closer to a charged elementary particle); between a transverse electromagnetic wave and the material continuum; between a longitudinal ether wave (dummy wave) and the material continuum.

1. INTRODUCTION

The development of Classical Electrodynamics in the late 19th and the beginning of 20th century ran into serious trouble from which Classical Electrodynamics was not able to recover (see R. Feynman's *Lectures on Physics*, vol.2, chapter 28)[1]. According to R. Feynman, this development "ultimately falls on its face" and "It is interesting, though, that the classical theory of electromagnetism is an unsatisfactory theory all by itself. There are difficulties associated with the *ideas* of Maxwell's theory which are not solved by and not directly associated with quantum mechanics." Further in the book he also writes: "To get a *consistent* picture, we must imagine that something holds the electron together." and "the extra nonelectrical forces are also known by the more elegant name "the Poincare stresses"". He then concludes: "-- there have to be other forces in nature to make a consistent theory of this kind." CED was discredited not only by R. Feynman but also by many other famous physicists. As a result, the whole of theoretical physics came to believe in the impossibility of explaining the stability of the electron’s charge by classical means, claiming responsibility on classical principles. But this is not true.

We showed earlier[2,3,4] and further elaborate here that there is nothing wrong with the basic classical *ideas* that Maxwell's theory is based upon. It simply needs further development. The work[2] opens the way to the normal (without singularities) development of CED. In this work it was shown that Poincare’s (1906) claim that the “material” part of an energy-momentum tensor (“Poincare stresses”) has to be of a “non-electromagnetic nature” (see Jackson[5]) is incorrect. It was given a definite material part expressed only through current desity (see formula (9) in[2]). It was given a static solution (Ideal Particle (IP), see (19)). The proper covariance of IP is a manifest. The charges actually hold togather and the energy inside IP comes from the inside electric field (positive energy) and the inside charge density (negative energy, see[2] formula (22)). The total energy inside IP is zero which means that the rest mass (total energy) correspond to the vacuum energy only. The contributions to the "inertial mass" (linear momentum divided on velocity) (R. Feynman called it "electromagnetic mass") can be calculated by making a Lorentz transformation and subsequent integration. The total inertial mass is equal to the rest mass (which is in complience with covariance) but the contributions are different: 4/3 comes from the vacuum electric field, 2/3 comes from the inside electric field, and –1 comes from the inside charge density. This is the explanation of the "anomalous factor of 4/3 in the inertia" (first found by J. J. Thomson in 1881)[5].

Let us begin with Maxwell's equations:

$$j^i + \frac{c}{4\pi} F^{ik}{}_{|k} = 0; \quad j^k{}_{|k} = 0 \tag{1}$$

$$div\vec{E} = \frac{4\pi}{c} j^0; \quad \oint_S \vec{E}\cdot d\vec{S} = \frac{4\pi}{c}\int_V j^0 dV \tag{1a}$$

$$rot\vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\dot{\vec{E}}; \quad \oint_\Gamma \vec{H}\cdot d\vec{\Gamma} = \frac{1}{c}\int_S \left(4\pi\vec{j} + \dot{\vec{E}}\right)\cdot d\vec{S} \tag{1b}$$

$$\frac{1}{c}\dot{j}^0 + div\vec{j} = 0; \quad \frac{\partial}{c\partial t}\int_V j^0 dV = \oint_S \vec{j}\cdot d\vec{S} \tag{1c}$$

The other half of Maxwell's equations is:

$$F^{*ik}{}_{|k} = 0; \quad F^{*ik} \equiv \frac{1}{2} e^{iklm} F_{lm} \tag{2}$$

$$div\vec{H} = 0; \quad \oint \vec{H}\cdot d\vec{S} = 0 \tag{2a}$$

$$rot\vec{E} = -\frac{1}{c}\dot{\vec{H}}; \quad \oint_\Gamma \vec{E}\cdot d\vec{\Gamma} = -\frac{1}{c}\int_S \dot{\vec{H}}\cdot d\vec{S} \tag{2b}$$

The equations are given in 4-d form, 3-d form, and in an integral form. Equation (1) represents the interaction law between the electromagnetic field and the current density. Equation (2) applies only to the electromagnetic field. This whole system (equation (1c) is not included) is definite for the 6 unknown components of the electromagnetic field on the condition that the currents (all the components) are given. This is the first order PDE system, the characteristics of which are the wave fronts.

What kind of currents can be given for this system? Not only continuous fields of currents can be prescribed. A jump in a current density is a normal situation. We can even go further and prescribe infinite (but the space integral has to be finite) current density. But in this case we have to check the results. In other words, the system allows that the given current density can contain Dirac's delta-functions if none of the integrals in (1), (2) goes infinite. But this is not the end. There exists an energy-momentum tensor that gives us the energy density in space. The space integral of that density also has to be finite. Here raises the problem. If we prescribe a point charge (3-d delta-function) then the energy integral will be infinite. If we prescribe a charged infinitely thin string (2-d delta-function) then the energy also will be infinite. But if we prescribe an infinitely thin surface with a finite surface charge density on it (1-d delta-function) then the energy integral will be finite. It looks like this is the only case that we can allow. But we have to remember that it is possible that a **disruption surface** (where the charge/current density can be infinite) can be present in our physical system. This kind of surface allows the electromagnetic field to have a jump across it (this very important fact was ignored in conventional CED – see below). It is also very important to understand that all these delta-functions for the charge distribution are at our discretion: we can prescribe them or we can "hold out". If we choose to prescribe ,then we are taking on additional responsibility. The major attempt to discredit CED (this is to remove any "obstacles" in the way of a quantum theory) was right here. The perpetrators of CED (including big names like R. Feynman in the USA and L. D. Landau in Russia but, remarkably, not A. Einstein) tried to convince us that a point charge is inherent to CED. With it comes the divergence of energy and the radiation reaction problem. This problem is solvable for the extended particle (which has infinite degrees of freedom) but is not solvable for the point particle. This is not an indication that "classical theory of electromagnetism is an unsatisfactory theory by itself". This rather means: do not use the point charge model (or charged string model). Only a charged closed surface model is usable.

We have another serious problem in conventional electrodynamics. As we have shown below, the variation procedure of conventional CED results in the requirement that the electromagnetic field must be continuous across any disruption surface. That actually means the impossibility of a surface charge/current on a disruption surface. I changed the variation procedure of CED and arrived at a theory where the electromagnetic interaction (ultimately represented by the Maxwell's equation (1)) is the only interaction. The so-called interaction term in Lagrangian ($A^k j_k$) is abandoned. So is abandoned the possibility of introducing any other interactions (like "strong" or "week"). I believe strongly that all the experimental data about elementary particles, quantum phenomena, and gravitation can be explained starting only with the electromagnetic interaction (1).

What is the right expression for the energy-momentum tensor that corresponds to the system of (1), (2)? Classical principles require that this expression must be unique. Conventional electrodynamics provides us with the expression: $T^{ik} = \mu c u^i u^k \dfrac{ds}{dt}$ (for the "material" part containing free particles only: see Landau[6] formula (33.5)) that contains density of mass (μ) and velocity only. No charge/current density is included. It seems that the mere presence of charge/current density has to contribute to the energy of the system. To correct the situation, we took the simplest possible Lagrangian with charge density:

$$\Lambda = -\frac{1}{16\pi} g^{ab} g^{cd} F_{ac} F_{bd} - \frac{2\pi}{k_0^2 c^2} g^{ab} j_a j_b \tag{3}$$

where k_0 is a new constant. No interaction term (like $A_k j^k$) is included.

2. VARIATION OF METRICS

Let us find the energy-momentum tensor that corresponds to the Lagrangian (3). The metric tensor in classical 4-space is g_{ik}=diag[1,-1,-1,-1] (we assume c=1). Let us consider an arbitrary variation of a metric tensor but on the condition that this variation does not introduce any curvature in space. This variation is:

$$\delta g_{ik} = \xi_{i|k} + \xi_{k|i} \tag{4}$$

where ξ^k is an arbitrary but small vector. One has to use the mathematical apparatus of General Relativity to check, that with the variation (3), the Riemann curvature tensor remains zero in the first order. Assuming that the covariant components of the physical fields are kept constant (then the contravariant components will be varied as a result of the variation of the metric tensor, but we do not use them -- see (3)) we can calculate the variation of the action. The variation of the square root of the determinant of the metric tensor is: $\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}\, g_{ik}\delta g^{ik}$ (this result can be found in textbooks on field theory). The variation of action becomes:

$$\delta S = -\int \left\{ 2\frac{\partial \Lambda}{\partial g^{ik}} - \Lambda g_{ik} \right\} \xi^{i|k} \sqrt{-g}\, d\Omega = -\int T_{ik} \xi^{i|k} \sqrt{-g}\, d\Omega$$
$$T_{ik} = -\frac{1}{4\pi} g^{ab} F_{ia} F_{kb} + \frac{1}{16\pi} F_{ab} F^{ab} g_{ik} - \frac{4\pi}{k_0^2} j_i j_k + \frac{2\pi}{k_0^2} j_a j^a g_{ik} \tag{5}$$

If our system consists of two regions that are separated by a closed disruption surface S then the above procedure has to be applied to each region separately. We can write: $T_{ik}\xi^{i|k} = (T_{ik}\xi^i)^{|k} - T_{ik}{}^{|k}\xi^i$. The 4-d volume integrals over divergence (the first term) can be expressed through 3-d hypersurface integrals according to a 4-d Gauss theorem. The integral over some remote closed surface reduces to zero due to the smallness of T_{ik} on infinity (usually assumed). The integral over the 3-d volume at t_1 and t_2 reduces to zero due to the assumption: $\xi_i = 0$ at these times. What is left is:

$$\delta S = -\int T_i^k \xi^i{}_{|k} \sqrt{-g}\, d\Omega = \int_S \left(T_i^k{}_{out} - T_i^k{}_{in} \right) \xi^i dS_k + \int_{in} T_i^k{}_{|k} \xi^i \sqrt{-g}\, d\Omega + \int_{out} T_i^k{}_{|k} \xi^i \sqrt{-g}\, d\Omega .$$

Since ξ_i are arbitrary small functions (between t_1 and t_2), the requirement $\delta S=0$ yields:

$$T^{ia}{}_{|a} = 0 \tag{6}$$

This condition has to be fulfilled in the inside and outside regions separately. And the additional requirement on the disruption surface S:

$$T^{ia} N_a \quad continuous \tag{6a}$$

where N_k is a normal to the surface.

We have found the unique definition of the energy-momentum tensor (5). If we want action to be minimum with respect to the arbitrary variation of the metric tensor in flat space then (6) and (6a) should be satisfied. Let us rewrite the energy-momentum tensor in 3-d form:

$$\begin{aligned}
T^{00} &= \frac{1}{8\pi}(E^2 + H^2) - \frac{2\pi}{k_0^2 c^2}[(j^0)^2 + (\vec{j})^2] \\
T^{11} &= \frac{1}{8\pi}(E^2 + H^2 - 2E_1^2 - 2H_1^2) - \frac{2\pi}{k_0^2 c^2}[(j^0)^2 - (\vec{j})^2 + 2(j^1)^2] \\
T^{01} &= \frac{1}{4\pi}(E_2 H_3 - E_3 H_2) - \frac{4\pi}{k_0^2 c^2} j^0 j^1 \\
T^{12} &= -\frac{1}{4\pi}(E_1 E_2 + H_1 H_2) - \frac{4\pi}{k_0^2 c^2} j^1 j^2
\end{aligned} \tag{5a}$$

Notice that we did not used Maxwell's or any other field equations so far. Also it should be noticed that the energy-momentum tensor (5), (5a) is not defined on the disruption surface itself despite of the fact that it can be a surface charge/current on a surface (infinite volume density but finite surface density).

Going further, **we are definitely stating that Maxwell's equation (1) is a universal law that should be fulfilled in all space without exceptions. It defines the interaction between the electromagnetic field and the field of current density. This law can not be subject to any variation procedure.** Maxwell's equation (2), we will confirm later as the result of a variation (see formula (9)). Substituting (5) in (6) and using Maxwell's equation (1) and antisymmetry of F_{ik}, we obtain:

$$j^a\left(\frac{k_0^2 c}{4\pi}F_{ai} + j_{a|i} - j_{i|a}\right) = 0, \quad j^0\left(\frac{k_0^2 c}{4\pi}\vec{E} + \nabla j^0 + \frac{1}{c}\dot{\vec{j}}\right) + \vec{j}\times\left(\frac{k_0^2 c}{4\pi}\vec{H} - rot\vec{j}\right) = 0 \qquad (7)$$

This equation has to be fulfilled in the inside and outside regions separately because the (6) is fulfilled separately in these regions. This is important. It is also important to realize that while the conservation of charge is fulfilled everywhere including a disruption surface – the disruption surface itself is an exempt from the energy-momentum conservation (no surface energy, no surface tension). This arrangement is in agreement with the fact that we can integrate delta-function (charge) but we can not integrate its square (would be energy).

3. A NEW DYNAMICS

Equation (7) we can call a **Dynamics Equation.** It is a nonlinear equation. But it has to be fulfilled inside and outside the particle separately. This will allow us to reduce it to a linear equation inside these regions. **Definition: vacuum is a region of space where all the components of current density are zero.** Equation (7) is automatically satisfied in vacuo, ($J^k=0$). And another possibility ($J^k\neq 0$) will be the inside region of elementary particle. The boundary between these regions will be a disruption surface. Inside the particle instead of (7) we have:

$$\frac{k_0^2 c}{4\pi}F_{ai} + j_{a|i} - j_{i|a} = 0; \quad \frac{k_0^2 c}{4\pi}\vec{E} + \nabla j^0 + \frac{1}{c}\dot{\vec{j}} = 0; \quad \frac{k_0^2 c}{4\pi}\vec{H} - rot\vec{j} = 0 \qquad (7a)$$

All the solutions of equation (7a) are also the solutions of nonlinear equation (7). At present, we know nothing about the solutions of (7) that do not satisfy (7a). Inside the elementary particle, the dynamics equation (7) or (7a) describes, as we call it, the **Material Continuum.** A Material Continuum can not be divided into a system of material points. The Relativistic (or Newton's) Dynamics Equation of CED, that describes the behavior of the particle as a whole, completely disappears inside the elementary particle. There is no mass, no force, no velocity or acceleration inside the particle. The field of current density jk defines a kinematics state of the Material Continuum. A world line of current j^k is not a world line of a material point. That allows us to deny any causal connection between the points on this line. as a consequence, j^k can be space-like as well as time-like. There is in no contradiction with the fact that the boundary of the particle can not exceed the speed of light. Equation (7a) is linear and allows superposition of different solutions. Using (1) we can obtain:

$$j^{k|a}{}_{|a} - k_0^2 j^k = 0; \quad \Box j^k + k_0^2 j^k = 0; \quad \Delta j^k - \frac{1}{c^2}\ddot{j}^k + k_0^2 j^k = 0 \qquad (7b)$$

By equation (7), we have gotten something very important, but we are just at the beginning of a difficult and uncertain journey. Now the current density can not be prescribed arbitrary. Inside the particle it has to satisfy equation (7b). Although there are no provisions on the surface current density, (if a surface current is different from zero then its density is necessarily expressed by a delta-function across the disruption surface).

4. THE ELECTROMAGNETIC POTENTIAL

Now we are going to vary the electromagnetic field F_{ik} in all space including a disruption surface. As usual, the variation is kept zero at t1 and t2 and also on a remote closed surface at infinity. In this case, the results of the variation will be in force on the disruption surface itself. Still, we have to write the variation formulas for each region separately. We claim that equation (1) can not be subject to variation. It is the preliminary condition before any variation. In our system, we have 10 unknown independent functions (4 functions in J_k and 6 functions in F_{ik}). These functions already have to satisfy 8 equations (4 equations in (1) and 4 equations in (7)). We have only 2 degrees of freedom left. We can not vary F_{ik} by a straightforward procedure. Let us employ here the Lagrange method of indefinite factors. Let us introduce a modified Lagrangian:

$$\Lambda' = \Lambda + A^a \left(j_a + \frac{1}{4\pi} F_{ab}{}^{|b} \right) \tag{8}$$

where A^k are 4 indefinite Lagrange factors. Now we have 2+4=6 degrees of freedom and we use them to vary F_{ik}. We have:

$$\delta S = -\int \left\{ \frac{\partial \Lambda'}{\partial F_{ik}} \delta F_{ik} + \frac{\partial \Lambda'}{\partial F_{ik|l}} \delta F_{ik|l} \right\} dV_4 = -\int \left\{ \left(\frac{\partial \Lambda'}{\partial F_{ik|l}} \delta F_{ik} \right)_{|l} + \left[\frac{\partial \Lambda'}{\partial F_{ik}} - \left(\frac{\partial \Lambda'}{\partial F_{ik|l}} \right)_{l} \right] \delta F_{ik} \right\} dV_4 = 0$$

The first term under integration is divergent and can be transformed to the hypersurface integral according to Gauss' theorem. Since the variation is arbitrary, the square brackets term has to be zero in either case. It gives:

$$F_{ik} = A_{k|i} - A_{i|k} \tag{9}$$

If V_4 is the inside region of the particle from t1 to t2 then the hypersurface integrals at t1 and t2 will be zero, but the hypersurface integral over the closed disruption surface will be:

$$\frac{1}{4\pi} \int dt \oint \left(A^i g^{kl} - A^k g^{il} \right)_{in} \delta F_{ik} dS_l \ .$$

If V_4 is the outside vacuum, then the hypersurface integrals at t1 and t2 will be zero. The hypersurface integral over the remote closed surface will be zero, but the hypersurface integral over the disruption surface will be:

$$-\frac{1}{4\pi} \int dt \oint \left(A^i g^{kl} - A^k g^{il} \right)_{out} \delta F_{ik} dS_l \ .$$

These integrals will annihilate if the **potential Ak is continuous across the disruption surface**. The continuity of potential does not preclude the possibility of a surface charge/current and a jump of the electromagnetic field as a consequence.

Claim: The variation procedure of conventional CED results in the impossibility of a surface charge/current on a disruption surface. The variation procedure of conventional CED starts up with equation (9) replacing the electromagnetic field with a potential. It introduces the interaction term $A^k j_k$ in the Lagrangian and varies the potential δA^k . As a result of least action, it obtains Maxwell's equation (1). But it can be shown that the consideration of a disruption surface will produce the requirement of electromagnetic field continuity. This actually denies the possibility of a single layer surface charge/current (the double layers are not interesting and they will require a jump of potential and an infinite electromagnetic field). Therefore, the conventional variation procedure is incorrect.

5. THE PHYSICAL MEANING OF POTENTIAL

Now we learned that the electromagnetic potential which was devoid of a physical meaning, has to be continuous across all the boundaries of disruption. This is a very important result. It allows me to reinterpret the physical meaning of potential. It is true that according to (9), we can add to the potential a gradient of some arbitrary function and the electromagnetic field won't change (gauge invariance). Yes, but this fact can be given another interpretation: **the potential is unique and it actually contains more information about physical reality than the electromagnetic field does.** To make the potential mathematically unique besides initial data and boundary conditions we need only to impose the conservation equation (formerly Lorenz gauge).

$$A^k{}_{|k} = 0; \quad A^{k|a}{}_{|a} = \frac{4\pi}{c} j^k; \quad \Box A^k = -\frac{4\pi}{c} j^k \qquad (10).$$

This is true everywhere. Using (1), (7a), and (9), we can conclude that inside a material continuum the potential has to satisfy:

$$(A^{k|b}_{\ |b} - k_0^2 A^k)^{|i} - (A^{i|b}_{\ |b} - k_0^2 A^i)^{|k} = 0 \qquad (7c)$$

If equation $$A^{k|b}_{\ |b} - k_0^2 A^k = 0; \quad or \quad \Box A^k + k_0^2 A^k = 0; \quad \Box \equiv \Delta - \frac{\partial^2}{c^2 \partial t^2} \qquad (11)$$

is satisfied, then (7c) is also satisfied. This type of equation is satisfied by the current density (see (7b)). This equation can be called a "Generalized Helmholtz Equation". In static (11) it coincides with Helmholtz equation. Equation (11) differs from the Klein-Gordon equation by the sign in front of the square of a constant.

The new interpretation of potential: A^0 represents the ether quantity (positive or negative), the 3-vector **A** represents the ether current. All together, the potential uniquely describes the existing physical reality – the ether. In general, the interpretation of potential doubles the interpretation of current.

6. THE IMPLICATIONS OF THE REINTERPRETATION OF POTENTIAL

Let us suppose that the potential is equal to a gradient of some function G, that we call "dummy generator":

$$A^k = g^{ka} G_{|a}; \quad A_0 = \frac{1}{c}\dot{G}; \quad \vec{A} = -\nabla G; \quad G^{|a}{}_{|a} = 0; \quad \Delta G - \frac{1}{c^2}\ddot{G} = 0 \qquad (12)$$

G has to be the solution of a homogeneous wave equation, though there are not any requirements for G on a disruption surface that we know of at present. But now we won't say that G is devoid of physical meaning (remember the mistake we made with potential).

What kind of physical process is described here by the corresponding potential? There is no electromagnetic field and the energy-momentum tensor is equal to zero. These are the "dummy waves" -- the longitudinal ether waves. These waves are physically significant only due to the boundary conditions on the disruption surfaces, which they affect**.** If this is the case, then G can be significant in physical experiments. It can even be unique under the laws (these laws are not completely clear) of another physical realm (the realm of electromagnetic potential).

It is difficult to imagine an elementary particle without some oscillating electromagnetic field inside it. If we assume that the oscillating field is present inside the particle, then the boundary conditions may require the corresponding oscillating electromagnetic field in the vacuum that surrounds the particle. It is easy to show that the energy of this vacuum electromagnetic field will be infinite. Though, it is possible that in the vacuum, only waves of scalar potential take care of the necessary boundary conditions. Since the potential is not present in the energy-momentum tensor (5), there won't be any energy connected to its presence. **We are free to suggest that the massive elementary particles are the sources of these waves**. These waves are emitted continuously with the amplitude (or its square) that is proportional to the mass of the particle (this proposition seems to be reasonable). These waves are only outgoing waves. The incoming waves can only be plane incoherent waves (spherical incoming coherent waves are impossible). We are not considering any incoming waves at this point.

First, we are going to show examples that the concept of material continuum really works.

7. OBTAINING SOLUTIONS

We are pleased that all the equations for finding the solutions are linear. That allows us to seek a total solution as a superposition of the **particular solutions** which satisfy to the equations and to the boundary conditions separately. The only non-linear condition is (6a) that has to be fulfilled only on the disruption surface. Only the total solution can be used in (6a).

IP2 (Ideal Particle Second): Let us obtain a simplest static spherical symmetric solution with electric charge and electric field only. We have:

$$\begin{aligned}
&A^0_{in} = \alpha\left(R_0(z) - R_0(z_1) + bz_1\right), \quad 0 \le z \le z_1; \quad j^0 = \frac{k_0^2 c}{4\pi}\alpha R_0(z) \\
&A^0_{out} = \alpha b\frac{z_1^2}{z}, \quad z_1 \le z < \infty; \quad b \equiv \sqrt{R_0^2(z_1) + R_1^2(z_1)}; \quad z = k_0 r \\
&E^r_{in} = \alpha k_0 R_1(z), \quad 0 \le z \le z_1; \quad E^r_{out} = \alpha k_0 b\frac{z_1^2}{z^2}, \quad z_1 \le z < \infty \\
&Q_{tot} = \frac{\alpha}{k_0} z_1^2 b; \quad Q_{surf} = \frac{\alpha}{k_0} z_1^2\left(b - R_1(z_1)\right) \\
&mc^2 = \frac{\alpha^2}{2k_0}\left(-z_1^2 R_0(z_1)R_1(z_1) + z_1^3 R_0^2(z_1) + z_1^3 R_1^2(z_1)\right) = \frac{\alpha^2}{2k_0}\left(z_1 - \sin(z_1)\cos(z_1)\right)
\end{aligned} \tag{12}$$

where $R_0(z)$ and $R_1(z)$ are spherical Bessel functions. In general, the electric field has a jump at the boundary of IP2. The position of boundary z_1 is arbitrary, but only at $z_1=n\pi$ (correspond to IP1) the surface charge is zero and the electric field is continuous. The first term in the mass expression (with the minus sign) corresponds to the energy of the inside region of the particle. It can be positive or negative depending on z_1 (at $z_1=n\pi$ it is zero). The second and third terms together represent the vacuum energy, which is positive. The total energy/mass remains positive at all z_1.

8. THE MECHANISM OF INTERACTION BETWEEN A CONSTANT ELECTRIC FIELD AND A STATIC CHARGE (SIMPLIFIED THIN LAYER MODEL)

The simplest solutions can be obtained in plane symmetry where all physical quantities depend only on the third coordinate – z. Let us consider symmetry of the type: vacuum – material continuum- vacuum. The thin layer of material continuum from z=0 to z=a (a is of the order of the size of an elementary particle) will represent a simplified model of an elementary particle. The boundaries at z=0 and z=a are deemed to be enforced by the particle and all the deficit of energy or momentum on these boundaries is deemed to go directly to the particle. Actually, if we have a deficit of energy or momentum, that means we are missing a particular solution that brings this deficit to zero (according to (6a)).

For further discussion we need to write down the integral form of the energy-momentum conservation:

$$\frac{\partial}{c\partial t}\int_V T^{m0}dV = -\oint_\Sigma T^{mq}d\Sigma_q \tag{6b}$$

where the V is a 3-d volume (which is not moving – it is our choice), and the Σ is a 3-d closed surface around this volume (obviously also not moving). The index m can correspond to any coordinate, while the index q corresponds only to the terrestrial coordinates (1,2,3). If m=0 then the left part of (6b) is the time rate of increasing of the energy inside V. The T^{0q} is a 3-dimensional Pointing vector (or the flow of energy through the unit of square per the unit of time). If m=3 (in the plane symmetry, only one coordinate is of interest) then the left part of (6b) is the time rate of increasing of the linear momentum of the volume V (actually it is a force applied to volume V). T^{3q} is the 3-vector (in general q can be 1,2,3. In our case, q=3) of the flow of linear momentum through the unit of the square per the unit of time. It is obvious that in static (or in a steady state) the left part of (6b) must be zero if there is no source/drain of energy/linear momentum inside the said volume. Suppose the constant electric field in the first vacuum region is E. The scalar potential (ether quantity), the electric field, and the charge density are:

$$\begin{gathered}\Phi_1 = -Ez + C_1;\quad E_1 = E;\quad \Phi_2 = -\frac{E}{k_0}\sin k_0 z + C_1\cos k_0 z;\quad C_1 = \frac{4\pi Q + E(1-\cos k_0 a)}{k_0\sin k_0 a}\\ E_2 = E\cos k_0 z + k_0 C_1 \sin k_0 z;\quad \rho = \frac{k_0^2}{4\pi}\Phi_2;\quad \Phi_3 = -(E+4\pi Q)(z-a) + C_2\\ C_2 = \frac{4\pi Q\cos k_0 a - E(1-\cos k_0 a)}{k_0\sin k_0 a};\quad E_3 = E + 4\pi Q\end{gathered} \tag{14}$$

Here, the charge density is the solution of (7b) inside the second region. The potentials are the solutions of (10). All the physical quantities except ρ are continuous on the boundaries. That means that the jumps of the components of the energy-momentum tensor will be due to the jumps of the charge density only. The energy momentum tensor in this symmetry (and this particular case) is:

$$T^{00} = \frac{1}{8\pi}E^2 - \frac{2\pi}{k_0^2}\rho^2 = T^{11} = T^{22};\quad T^{03} = 0;\quad T^{33} = -\frac{1}{8\pi}E^2 - \frac{2\pi}{k_0^2 c}\rho^2 \tag{15}$$

There is no energy flow in this system. But there is the flow of linear momentum. In the first vacuum region, it is: $T^{33}=-E^2/8\pi$. Then it jumps on the first and on the second boundaries:

$$T^{33}(z=0+)-T^{33}(z=0-)=-\frac{k_0^2C_1^2}{8\pi};\quad T^{33}(z=a+)-T^{33}(z=a-)=\frac{k_0^2C_2^2}{8\pi}$$
$$\frac{k_0^2C_1^2}{8\pi}-\frac{k_0^2C_2^2}{8\pi}=Q\left(E+\frac{4\pi Q}{2}\right) \tag{16}$$

After that, it is: $T^{33}=-(E+4\pi Q)^2/8\pi$. As we go from left to right, the jump on the first boundary is negative. That means that the small volume that includes the first boundary gets negative outside (we always consider the outside normal to the closed surface Σ) the flow of linear momentum. That means that the volume itself (according to the formula (6b)) gets a positive rate of linear momentum, which is the force in the positive direction of the z-axis. The first boundary is pushed in the positive direction of the z-axis. The second boundary is also pushed, but in the negative direction of the z-axis. The difference is exactly equal to the force with which the field acts on a particle (see (16)). We see that electric field does not act on a charge *per se,* but only on a whole particle and only through its boundaries. This picture is true only at t=0, because the missing particular solution that makes the appearance of “free” sources and drains, most definitely will depend on time (the particle will begin to accelerate). This is the actual success of the proposed modification of CED.

9. THE MECHANIZM OF INTERACTION BETWEEN A CONSTANT ELECTRIC FIELD AND A STSTIC CHARGE (SPHERICAL CHARGE)

Here,we will confirm that the thin layer treatment corresponds to the more accurate but more complicated spherical charge treatment. Suppose we have a constant electric field E, directed along the z-axis in the vacuum. Also we have a sphere of r_1-radius that separates the material continuum inside the sphere from the vacuum. The situation is static at t=0. The potential in general, has to satisfy the equation $A^k{}_{|k}=0$ (10) everywhere, and equation (7c) inside the material continuum. This last equation with 3rd derivatives, has to be satisfied strictly inside a material continuum and not on the disruption surface itself (where a single layer of charge/current density is possible and the charge/current density ($j^k=\frac{c}{4\pi}A^{k|a}{}_{|a}$) can be infinite).

If the equation
$$A^{k|b}{}_{|b}-k_0^2A^k=0;\quad or\quad \Box A^k+k_0^2A^k=0;\quad \Box\equiv\Delta-\frac{\partial^2}{c^2\partial t^2} \tag{17}$$

Is satisfied, then (7c) is also satisfied. This equation can be called the “Generalized Helmholtz Equation”. The static (17) coincides with Helmholtz’s equation . Equation (17) differs from the Klein-Gordon equation by the sign in front of the square of a constant. In the vacuum:

$$A^{k|a}{}_{|a}=0 \tag{18}$$

The general solution in the whole space can be the sum of a number of particular solutions. The potential of each particular solution has to be continuous on a disruption surface. But the conservation condition: “continuity of a vector $T^{ik}N_k$ ” has to be fulfilled only for the general solution. Let us define a “dummy” potential by:

$$D^k{}_{|k}=0,\quad D^{i|k}-D^{k|i}=0,\quad consequently:\quad D^{k|a}{}_{|a}=0$$

If we have a solution A^k of (10)+(7c) or a solution of (10)+(18) then A^k+D^k will also be the solution of the same equations (it does not matter if inside the material continuum or in the vacuum).

Now we return to our particular case. The solution of (18) that we would be interested in: D^0=const. If there is no time dependence then (10) is satisfied for any A^0 if a vector potential is zero. Equation (7c) is Laplace's operator taken from a Helmholtz equation. The solutions of the Helmholtz equation that we consider would be: $R_0(k_0r)$ and $R_1(k_0r)\cos(\theta)$ where $R_n()$ are the spherical Bessel functions. In the vacuum, we consider the solutions: e/r where e is the total charge, $r\cos(\theta)$, and $(1/r^2)\cos(\theta)$ So, let us consider the potential:

$$\begin{aligned} A^0{}_{in} &= \alpha R_0(k_0 r) + \frac{e}{r_1} - \alpha R_0(k_0 r_1) \\ A^0{}_{out} &= \frac{e}{r} + E\left(\frac{r_1^3}{r^2} - r\right)\cos(\theta) \end{aligned} \tag{19}$$

It is continuous at $r=r_1$. The corresponding electric field and charge density will be:

$$\begin{aligned} &E_{rin} = \alpha k_0 R_1(k_0 r); \quad E_{rout} = \frac{e}{r^2} + E\left(1 + 2\frac{r_1^3}{r^3}\right)\cos(\theta) \\ &E_{\theta in} = 0; \quad E_{\theta out} = E\left(\frac{r_1^3}{r^3} - 1\right)\sin(\theta); \quad \rho = \frac{\alpha k_0^2}{4\pi} R_0(k_0 r) \end{aligned} \tag{20}$$

We see that the radial component of the electric field has a jump while the θ component is continuous. The surface charge density and the total surface charge are:

$$\begin{aligned} &4\pi\rho_{surf} = -E_{rin}(r_1) + E_{rout}(r_1) = \frac{e}{r_1^2} - \alpha k_0 R_1(k_0 r_1) + 3E\cos(\theta) \\ &Q_{surf\,tot} = e - \alpha k_0 r_1^2 R_1(k_0 r_1) \end{aligned} \tag{21}$$

We see that it does not matter what the relation between the constants α and e is, the surface of the particle has a "surface charge polarization" $3E\cos(\theta)$. Only this polarization will result in the net force on the charge. The polarization in the volume of the particle can be introduced using the solution $R_1(k_0r)\cos(\theta)$. But this polarization won't change the net force (it can be introduced with any constant factor). We've made the corresponding calculations that supports this statement. We do not present them here for simplification.

The double radial component of energy-momentum tensor will be:

$$\begin{aligned} &8\pi T^{rr} = E_\theta^2 - E_r^2 - \frac{16\pi^2}{k_0^2}\rho^2; \quad 8\pi T^{rr}_{surf\,in} = -\alpha^2 k_0^2\left(R_0^2(k_0 r_1) + R_1^2(k_0 r_1)\right) \\ &8\pi T^{rr}_{surf\,out} = -\left[\frac{e^2}{r_1^4} + \frac{6e}{r_1^2}E\cos(\theta) + 9E^2\cos^2(\theta)\right]; \quad T^{\theta r}_{surf\,in} = T^{\theta r}_{surf\,out} = 0 \end{aligned} \tag{22}$$

The force applied to the surface will be normal to the surface and equal to $T^{rr}_{surf\,in} - T^{rr}_{surf\,out}$. This force is zero if E=0. This case corresponds to the true static solution of our equations with (6a) satisfied. This solution enforces the spherical boundary. If E is not zero, then we do not know the actual solution because (6a) is not satisfied. The actual solution will be not static. But we can calculate the force at the moment when E was "turned on". To take the z component of this force we have to multiply the expression by cos(θ). If we integrate this over the sphere surface then all the terms except the one with cos(θ) produce zero. The result of integration will be eE. This is exactly the force with which the electric field E acts on a charge e.

10. THE TRANSVERSE ELECTROMAGNETIC WAVE

Let us consider that the transverse electromagnetic wave is coming from the left and encounters a layer of the material continuum. We expect to find transmitted and reflected waves as well as radiation pressure. "Behind" the transverse E-M wave we find a transverse ether wave with only the x component (for x-polarized E-M wave) of the vector potential (ether current) different from zero

$$
\begin{aligned}
&{}_1A^1 = \Phi_1^+ + \Phi_1^-; \quad \Phi_1^+ = F_1^+ e^{-ikz}; \quad \Phi_1^- = F_1^- e^{ikz}; \quad k = \frac{\omega}{c} \\
&{}_1E_1 = -ik \cdot {}_1A^1; \quad {}_1H_2 = -ik \cdot \left(\Phi_1^+ - \Phi_1^-\right); \quad (k')^2 = k_0^2 + k^2 \\
&{}_2A^1 = \Phi_2^+ + \Phi_2^-; \quad \Phi_2^+ = F_2^+ e^{-ik'z}; \quad \Phi_2^- = F_2^- e^{ik'z} \\
&{}_2E_1 = -ik \cdot {}_2A^1; \quad {}_2H_2 = -ik' \cdot \left(\Phi_2^+ - \Phi_2^-\right); \quad j(z,t) = \frac{ck_0^2}{4\pi} \cdot {}_2A^1 \\
&{}_3A^1 = F_3^+ e^{-ikz}; \quad {}_3E_1 = -ik \cdot {}_3A^1; \quad {}_3H_2 = -ik \cdot {}_3A^1
\end{aligned} \tag{23}
$$

where the prefixes to the fields always denote the number of the region (we did not supply the current density j with indexes because it is different from zero only in the second region). We assume that all the functions depend on t through the factor exp(iωt). In the first region, the given incoming wave F_1^+ and some reflected wave F_1^- are present. In the second region two waves are present. They satisfy the equations:

$$
{}_2A^1{}'' + k^2 \cdot {}_2A^1 = -\frac{4\pi}{c} j; \quad \frac{\partial}{\partial x} \cdot {}_2A^1 = 0 \tag{24}
$$

On the boundaries, the vector potential (ether current) and its first derivative have to be continuous. We found:

$$
\begin{aligned}
&F_1^- = -F_1^+ \frac{2ik_0^2 \sin(k'a)}{D}; \quad F_3^+ e^{-ika} = F_1^+ \frac{4kk'}{D} \\
&D \equiv (k+k')^2 e^{ik'a} - (k-k')^2 e^{-ik'a} \\
&F_2^+ = F_1^+ \frac{2k(k+k')}{D} e^{ik'a}; \quad F_2^- = F_1^+ \frac{2k(k'-k)}{D} e^{-ik'a}
\end{aligned} \tag{25}
$$

Here we found the amplitudes of reflected and transmitted waves and the amplitudes of both waves in the second region (only F_1^+ is considered to be real and given). We found previously that the energy-momentum tensor in a material continuum has the form (one-dimensional symmetry assumed):

$$
\begin{aligned}
&T^{00} = \frac{1}{8\pi}\left(E^2 + H^2\right) - \frac{2\pi}{k_0^2 c^2} j^2; \quad T^{33} = \frac{1}{8\pi}\left(E^2 + H^2\right) + \frac{2\pi}{k_0^2 c^2} j^2 \\
&T^{11} = \frac{1}{8\pi}\left(-E^2 + H^2\right) - \frac{2\pi}{k_0^2 c^2} j^2 = -T^{22}; \quad T^{03} = \frac{1}{4\pi} EH
\end{aligned} \tag{26}
$$

Since we use complex numbers – we have to take the real parts of the physical values, multiply them and then take the time average. The result will be the real part of the product of the first complex amplitude on the conjugate of the second complex amplitude. The result in the second region is:

$$
\begin{aligned}
&\frac{2\pi}{k_0^2c^2}j^2 = F_1^{+2}\frac{k_0^2k^2}{\pi|D|^2}\left(k_0^2+2k^2+k_0^2\cos 2k'(a-z)\right)\\
&T^{00} = -F_1^{+2}\frac{2k^2}{\pi|D|^2}\left(k_0^4\cos 2k'(a-z)-k^2(k_0^2+2k^2)\right)\\
&T^{03} = F_1^{+2}\frac{4k^4k'^2}{\pi|D|^2};\quad T^{33} = F_1^{+2}\frac{2k^2k'^2}{\pi|D|^2}\left(k_0^2+2k^2\right)
\end{aligned}
\tag{27}
$$

The electric and magnetic fields are continuous in this system. The flow of energy appear to be independent of z in the second region. It is continuous on the boundaries (see (26) that the currents are not included in T^{03}). That means that it is constant through the whole system. The flow of linear momentum (T^{33}) is positive in the first region and then jumps up on the first boundary due to the jump of the current j. It means that the surface integral in (6b) is positive and the first boundary is loosing linear momentum. The surface is pulled in the negative direction of the z-axis. But this pull is less than another pull due to the jump on the second boundary (this can be figured out from (27). We consider k'a~π/4, but it will be true for any k'a different from π. Notice also that at k'a=π, the reflected wave is zero as can be seen from (25)). As a result, the material continuum will experience the force (through its boundaries) in the positive direction of the z-axis. The numerical value of this force can be calculated from the jumps and it is equal to the force that we usually calculate from the linear momentum of incident transmitted and reflected waves.

11. THE LONGITUDINAL ETHER (DUMMY) WAVE

Let us consider that the longitudinal ether wave is coming from the left and encounters the layer of material continuum. There are no electromagnetic fields that accompany this wave in vacuum. Not so inside the material continuum. We have:

$$
\begin{aligned}
&{}_1A^0 = \Phi_1^+ + \Phi_1^-;\quad \Phi_1^+ = F_1^+e^{-ikz};\quad \Phi_1^- = F_1^-e^{ikz};\quad {}_1A^3 = \Phi_1^+ - \Phi_1^-\\
&{}_2A^0 = \Phi_2^+ + \Phi_2^-;\quad \Phi_2^+ = F_2^+e^{-ik'z};\quad \Phi_2^- = F_2^-e^{ik'z};\quad k = \frac{\omega}{c}\\
&{}_2A^3 = \frac{k}{k'}\left(\Phi_2^+ - \Phi_2^-\right);\quad j^0(z,t) = \frac{ck_0^2}{4\pi}\left(\Phi_2^+ + \Phi_2^-\right);\quad (k')^2 = k_0^2 + k^2\\
&j^3(z,t) = \frac{ck_0^2k}{4\pi k'}\left(\Phi_2^+ - \Phi_2^-\right);\quad E_3 = \frac{ik_0^2}{k'}\left(\Phi_2^+ - \Phi_2^-\right);\quad {}_3A^0 = {}_3A^3 = F_3^+e^{-ikz}
\end{aligned}
\tag{28}
$$

where we assume that all the functions depend on t through the factor exp(iωt). In the first region, the given incoming wave F_1^+ and some reflected wave F_1^- are present (both are dummy waves). In the second region, two waves are present. They satisfy to the equations:

$$
{}_2A^0{}'' + k^2\cdot{}_2A^0 = -\frac{4\pi}{c}j^0;\quad {}_2A^3{}'' + k^2\cdot{}_2A^3 = -\frac{4\pi}{c}j^3;\quad ik\cdot{}_2A^0 + {}_2A^3{}' = 0 \tag{29}
$$

To define all the waves, we have to satisfy the conditions on the boundaries. The scalar potential (ether quantity) and the vector potential (ether current) should be continuous across the boundaries. We found:

$$
\begin{aligned}
&on\ z=a: \quad F_2^+ e^{-ik'a} = \frac{k+k'}{2k} F_3^+ e^{-ika}; \quad F_2^- e^{ik'a} = -\frac{k'-k}{2k} F_3^+ e^{-ika} \\
&on\ z=0: \quad F_1^- = F_1^+ \frac{2ik_0^2 \sin(k'a)}{D}; \quad F_3^+ e^{-ika} = F_1^+ \frac{4kk'}{D} \\
&D \equiv (k+k')^2 e^{ik'a} - (k'-k)^2 e^{-ik'a} \\
&F_2^+ = F_1^+ \frac{2k'(k+k')}{D} e^{ik'a}; \quad F_2^- = -F_1^+ \frac{2k'(k'-k)}{D} e^{-ik'a}
\end{aligned} \tag{30}
$$

Here we found the amplitudes of reflected and transmitted waves and the amplitudes of both waves in the second region (only F_1^+ is considered to be real).

From (28) we can calculate the derivatives:

$$
{}_1A^0{}' = -ik \cdot {}_1A^3; \quad {}_2A^0{}' = -\frac{ik'^2}{k} \cdot {}_2A^3; \quad {}_1A^3{}' = -ik \cdot {}_1A^0; \quad {}_2A^3{}' = -ik \cdot {}_2A^0; \tag{28a}
$$

We see that the ether current (A^3) has a continuous derivative while the derivative of the ether quantity (A^0) has a jump on the boundaries. That means that there are surface charges associated with the boundaries.

We notice from (28) that the electric field, charge density, and current density are different from zero inside the second region. This means that the material continuum produces a kind of physical response to the energy-less dummy waves. We also found previously that the energy-momentum tensor in a material continuum has the form (one-dimensional symmetry assumed):

$$
\begin{aligned}
&T^{00} = \frac{1}{8\pi} E^2 - \frac{2\pi}{k_0^2 c^2}\left(c^2\rho^2 + j^2\right); \quad T^{11} = -\frac{1}{8\pi} E^2 - \frac{2\pi}{k_0^2 c^2}\left(c^2\rho^2 + j^2\right) \\
&T^{22} = T^{33} = \frac{1}{8\pi} E^2 - \frac{2\pi}{k_0^2 c^2}\left(c^2\rho^2 - j^2\right); \quad T^{01} = -\frac{4\pi}{k_0^2 c^2} c\rho j
\end{aligned} \tag{31}
$$

To actually calculate a time-average of the energy-momentum tensor we have to take the real parts of the physical values, multiply them and then take the time average. The result will be the real part of the product of the first complex amplitude on the conjugate of the second complex amplitude. The result of calculation is:

$$
\begin{aligned}
&T^{33} = -(F_1^+)^2 \frac{2k_0^4}{\pi |D|^2}(k_0^2 + 2k^2); \quad T^{03} = -(F_1^+)^2 \frac{4k_0^2 k^2 k'^2}{\pi |D|^2} \\
&T^{00} = (F_1^+)^2 \frac{2k_0^6}{\pi |D|^2} \cos(2k'(a-z))
\end{aligned} \tag{32}
$$

The first two time averages of the tensor components appear to be independent of z. The energy density depends on z. All these tensor components are zero in the both vacuum regions. That means that all of them jump on the boundaries.

On the first boundary, the jump of T^{33} is negative. It means that the first boundary will be pushed to the right. On the second boundary, the jump will be positive and of the same absolute value (because T^{33} is constant inside the second region). The second boundary will be pushed in the negative direction of the z-axis with the same force – we have equilibrium – no “free” force.

On the first boundary, the jump of T^{03} is negative. This means that the first boundary will be getting energy. On the second boundary, the jump will be positive and the same by its absolute value (because T^{03} is constant inside the second region). The second boundary will be losing the same amount of energy – no "free" energy.

It looks like in the particular solutions that we have carried energy and momentum from the second boundary to the first, while the missing particular solution carries them back.

At the present time we hesitate to proceed further because the meaning of these results still has to be clarified.

12. DE-BROGLIE'S WAVES

Let us suppose, in addition (see section 6), that the frequency of dummy waves (as well as the intensity) are also proportional to the mass of the particle: $\omega = mc^2/\hbar$. The motionless particles are present in abundance in the experimental arrangement itself. These particles can be partially synchronized in some proximity (the extent of this proximity is not yet known) of any point inside the experimental device. We can expect some standing scalar waves of a dummy generator that can be experienced by the moving particle independent of the direction of motion. In this case, we can explain De-Broglie's waves as beat frequency waves between the frequency of motionles particles and the Doppler shifted frequency of a moving particle. The role of non-linear device that is necessary to obtain the beat frequency wave can be very well played by the boundary of the particle itself. This will explain "the wave properties of particles" by pure classical means. These ideas were first expressed by Milo Wolf(7) in 1993.

Above, in the reformulation of conventional classical electrodynamics, we omitted the interaction term in the Lagrangian/Hamiltonian. Quantum Theory was undermined by this action. One can notice that historically, after the creation of quantum theory, there were attempts to legalize the electromagnetic potential as a physically measurable value (see R. Feynman(1)). Still, it is too early to try to find the classical basis for the quantum theory but the direction to go is in the physical realm of electromagnetic potential.

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