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On The Physical Meaning of The Energy-Momentum Tensor

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In the previous articles we have presented several solutions of the type: vacuum – material continuum- vacuum. The symmetry was plane (all physical quantities can depend only on the third coordinate – z). The results of these calculations pose some questions as to how to understand the meaning of the energy-momentum tensor. The integral form of the energy-momentum conservation is:

$$\frac{\partial}{c\partial t}\int_V T^{m0}dV = -\oint_\Sigma T^{mq}d\Sigma_q \qquad (1)$$

where the V is a 3-d volume (which is not moving – it is our choice), and the Σ is a 3-d closed surface around this volume (obviously also not moving). The index m can correspond to any coordinate, while the index q corresponds only to the terrestrial coordinates (1,2,3).

If m=0 then the left part of (1) is the time rate of increasing of the energy inside V. The T^{0q} is 3-d Pointing's vector (or the flow of energy through the unit of square per the unit of time).

If m=3 (in the plane symmetry only one coordinate is of interest) then the left part of (1) is the time rate of increasing of the linear momentum of the volume V (actually it is a force applied to the volume V). The T^{3q} is the 3-vector (in general q can be 1,2,3. In our case q=3) of the flow of linear momentum through the unit of square per the unit of time). It is obvious that in static (or in a steady state) the left part of (1) must be zero if there is no source/drain of energy/linear momentum inside the said volume. Let us now discuss the results of previous calculations.

1. **The charge in the static electric field:**

The energy momentum tensor in this symmetry (and this particular case) is:
2.

$$T^{00} = \frac{1}{8\pi}E^2 - \frac{2\pi}{k_0^2}\rho^2 = T^{11} = T^{22}; \quad T^{03} = 0; \quad T^{33} = -\frac{1}{8\pi}E^2 - \frac{2\pi}{k_0^2 c}\rho^2 \qquad (2)$$

The scalar potential (ether quantity), the electric field, and the charge density are:

$$\Phi_1 = -Ez + C_1; \quad E_1 = E; \quad \Phi_2 = -\frac{E}{k_0}\sin k_0 z + C_1\cos k_0 z; \quad C_1 = \frac{4\pi Q + E(1-\cos k_0 a)}{k_0 \sin k_0 a}$$

$$E_2 = E\cos k_0 z + k_0 C_1 \sin k_0 z; \quad \rho = \frac{k_0^2}{4\pi}\Phi_2; \quad \Phi_3 = -(E + 4\pi Q)(z-a) + C_2 \qquad (3)$$

$$C_2 = \frac{4\pi Q\cos k_0 a - E(1-\cos k_0 a)}{k_0 \sin k_0 a}; \quad E_3 = E + 4\pi Q$$

All the physical quantities except ρ are continuous on the boundaries. That means that the jumps of the components of the energy-momentum tensor will be due to the jumps of the charge density only.

There is no energy flow in this system. But there is the flow of linear momentum (I was incorrect in my previous article). In the first vacuum region it is: $T^{33}=-E^2/8\pi$. Then it jumps on the first and on the second boundaries:

$$T^{33}(z=0+)-T^{33}(z=0-)=-\frac{k_0^2C_1^2}{8\pi}; \quad T^{33}(z=a+)-T^{33}(z=a-)=\frac{k_0^2C_2^2}{8\pi}$$
$$\frac{k_0^2C_1^2}{8\pi}-\frac{k_0^2C_2^2}{8\pi}=Q\left(E+\frac{4\pi Q}{2}\right) \tag{4}$$

After that it is: $T^{33}=-(E+4\pi Q)^2/8\pi$. As we go from left to right the jump on the first boundary is negative. That means that the small volume that includes the first boundary gets negative outside (we always consider the outside normal to the closed surface Σ) flow of the linear momentum. That means that the volume itself (according to the formula (1)) gets the positive rate of linear momentum, which is the force in the positive direction of z-axis. The first boundary is pushed in the positive direction of z-axis. The second boundary is also pushed but in the negative direction of z-axis. The difference is exactly equal to the force with which the field acts on a charge.

2. The transverse electromagnetic wave.

This case was not presented before – so we have to present it now. Let us consider that the transverse electromagnetic wave is coming from the left and encounters the layer of material continuum. We expect to find the transmitted and reflected waves as well as the radiation pressure. Behind the transverse E-M wave we find the transverse ether wave with only x component (for x-polarized E-M wave) of the vector potential (ether current) is different from zero:

$$\begin{aligned}
&{}_1A^1=\Phi_1^+ + \Phi_1^-; \quad \Phi_1^+ = F_1^+ e^{-ikz}; \quad \Phi_1^- = F_1^- e^{ikz}; \quad k=\frac{\omega}{c} \\
&{}_1E_1=-ik\cdot {}_1A^1; \quad {}_1H_2=-ik\cdot\left(\Phi_1^+ - \Phi_1^-\right); \quad (k')^2=k_0^2+k^2 \\
&{}_2A^1=\Phi_2^+ + \Phi_2^-; \quad \Phi_2^+ = F_2^+ e^{-ik'z}; \quad \Phi_2^- = F_2^- e^{ik'z} \\
&{}_2E_1=-ik\cdot {}_2A^1; \quad {}_2H_2=-ik'\cdot\left(\Phi_2^+ - \Phi_2^-\right); \quad j(z,t)=\frac{ck_0^2}{4\pi}\cdot {}_2A^1 \\
&{}_3A^1=F_3^+ e^{-ikz}; \quad {}_3E_1=-ik\cdot {}_3A^1; \quad {}_3H_2=-ik\cdot {}_3A^1
\end{aligned} \tag{5}$$

where the prefixes to the fields always denote the number of the region (we did not supply with indexes the current density j because it is different from zero only in the second region). We assume that all the functions depend of t through the factor exp(iωt). In the first region the given incoming wave F_1^+ and some reflected wave F_1^- are present. In the second region two waves are present. They satisfy to the equations:

$${}_2A^1{}''+k^2\cdot {}_2A^1=-\frac{4\pi}{c}j; \quad \frac{\partial}{\partial x}\cdot {}_2A^1=0 \tag{6}$$

On the boundaries the vector potential (ether current) and its first derivative have to be continuous. We found:

$$F_1^- = -F_1^+ \frac{2ik_0^2 \sin(k'a)}{D}; \quad F_3^+ e^{-ika} = F_1^+ \frac{4kk'}{D}$$
$$D \equiv (k+k')^2 e^{ik'a} - (k-k')^2 e^{-ik'a} \tag{7}$$
$$F_2^+ = F_1^+ \frac{2k(k+k')}{D} e^{ik'a}; \quad F_2^- = F_1^+ \frac{2k(k'-k)}{D} e^{-ik'a}$$

Here we found the amplitudes of reflected and transmitted waves and the amplitudes of both waves in the second region (only F_1^+ is considered to be real and given).

We found previously that the energy-momentum tensor in a material continuum has the form (one-dimensional symmetry assumed):

$$T^{00} = \frac{1}{8\pi}\left(E^2 + H^2\right) - \frac{2\pi}{k_0^2 c^2} j^2; \quad T^{33} = \frac{1}{8\pi}\left(E^2 + H^2\right) + \frac{2\pi}{k_0^2 c^2} j^2$$
$$T^{11} = \frac{1}{8\pi}\left(-E^2 + H^2\right) - \frac{2\pi}{k_0^2 c^2} j^2 = -T^{22}; \quad T^{03} = \frac{1}{4\pi} EH \tag{8}$$

Since we use complex numbers – we have to take the real parts of the physical values, multiply them and then take the time average. The result will be equal to ½ multiplied on the real part of the product of the first complex amplitude on the conjugate of the second complex amplitude. The result in the second region is:

$$\frac{2\pi}{k_0^2 c^2} j^2 = F_1^{+2} \frac{k_0^2 k^2}{\pi |D|^2}\left(k_0^2 + 2k^2 + k_0^2 \cos 2k'(a-z)\right)$$
$$T^{00} = -F_1^{+2} \frac{2k^2}{\pi |D|^2}\left(k_0^4 \cos 2k'(a-z) - k^2(k_0^2 + 2k^2)\right) \tag{9}$$
$$T^{03} = F_1^{+2} \frac{4k^4 k'^2}{\pi |D|^2}; \quad T^{33} = F_1^{+2} \frac{2k^2 k'^2}{\pi |D|^2}\left(k_0^2 + 2k^2\right)$$

The electric and magnetic fields are continuous in this system. The flow of energy appear to be independent of z in the second region. It is continuous on the boundaries (see in (8) that the currents are not included in T^{03}). That means that it is constant through the whole system. The flow of linear momentum (T^{33}) is positive in the first region and then jumps up on the first boundary due to the jump of the current j. That means that the surface integral in (1) is positive and the first boundary is loosing linear momentum. The surface is pulled in the negative direction of z-axis. But this pull is less than another pull due to the jump on the second boundary (this can be figured out from (9). We consider k'a~π/4, but it will be true for any k'a different from π. Notice also that at k'a=π the reflected wave is zero as can be seen from (7)). As a result the material continuum will experience the force (through its boundaries) in the positive direction of z-axis. The numerical value of this force can be calculated from the jumps and it is equal to the force that we usually calculate from the impulses of incident transmitted and reflected waves.

3. The longitudinal ether (dummy) wave.

Let us consider that the longitudinal ether wave is coming from the left and encounters the layer of material continuum. There are no electromagnetic fields that accompany this wave in vacuum. Not so inside the material continuum. We have:

$$
\begin{aligned}
&{}_1A^0 = \Phi_1^+ + \Phi_1^-; \quad \Phi_1^+ = F_1^+ e^{-ikz}; \quad \Phi_1^- = F_1^- e^{ikz}; \quad {}_1A^3 = \Phi_1^+ - \Phi_1^- \\
&{}_2A^0 = \Phi_2^+ + \Phi_2^-; \quad \Phi_2^+ = F_2^+ e^{-ik'z}; \quad \Phi_2^- = F_2^- e^{ik'z}; \quad k = \frac{\omega}{c} \\
&{}_2A^3 = \frac{k}{k'}\left(\Phi_2^+ - \Phi_2^-\right); \quad j^0(z,t) = \frac{ck_0^2}{4\pi}\left(\Phi_2^+ + \Phi_2^-\right); \quad (k')^2 = k_0^2 + k^2 \\
&j^3(z,t) = \frac{ck_0^2 k}{4\pi k'}\left(\Phi_2^+ - \Phi_2^-\right); \quad E_3 = \frac{ik_0^2}{k'}\left(\Phi_2^+ - \Phi_2^-\right); \quad {}_3A^0 = {}_3A^3 = F_3^+ e^{-ikz}
\end{aligned} \tag{10}
$$

where we assume that all the functions depend of t through the factor exp(iωt). In the first region the given incoming wave F_1^+ and some reflected wave F_1^- are present (both are dummy waves). In the second region two waves are present. They satisfy to the equations:

$$
{}_2A^0{}'' + k^2 \cdot {}_2A^0 = -\frac{4\pi}{c} j^0; \quad {}_2A^3{}'' + k^2 \cdot {}_2A^3 = -\frac{4\pi}{c} j^3; \quad ik \cdot {}_2A^0 + {}_2A^3{}' = 0 \tag{11}
$$

To define all the waves we have to satisfy the conditions on the boundaries. The scalar potential (ether quantity) and the vector potential (ether current) should be continuous across the boundaries. We found:

$$
\begin{aligned}
&on\ z = a: \quad F_2^+ e^{-ik'a} = \frac{k+k'}{2k} F_3^+ e^{-ika}; \quad F_2^- e^{ik'a} = -\frac{k'-k}{2k} F_3^+ e^{-ika} \\
&on\ z = 0: \quad F_1^- = F_1^+ \frac{2ik_0^2 \sin(k'a)}{D}; \quad F_3^+ e^{-ika} = F_1^+ \frac{4kk'}{D} \\
&D \equiv (k+k')^2 e^{ik'a} - (k'-k)^2 e^{-ik'a} \\
&F_2^+ = F_1^+ \frac{2k'(k+k')}{D} e^{ik'a}; \quad F_2^- = -F_1^+ \frac{2k'(k'-k)}{D} e^{-ik'a}
\end{aligned} \tag{12}
$$

Here we found the amplitudes of reflected and transmitted waves and the amplitudes of both waves in the second region (only F_1^+ is considered to be real).

From (10) we can calculate the derivatives:

$$
{}_1A^0{}' = -ik \cdot {}_1A^3; \quad {}_2A^0{}' = -\frac{ik'^2}{k} \cdot {}_2A^3; \quad {}_1A^3{}' = -ik \cdot {}_1A^0; \quad {}_2A^3{}' = -ik \cdot {}_2A^0; \tag{10a}
$$

We see that the ether current (A^3) has continuous derivative while the derivative of ether quantity (A^0) has jump on the boundaries. That means that there are surface charges associated with the boundaries.

We notice from (10) that the electric field, charge density, and current density are different from zero inside the second region. That means that the material continuum produces kind of a physical response to the energy-less dummy waves. We also found previously that the energy-momentum tensor in a material continuum has the form (one-dimensional symmetry assumed):

$$
\begin{aligned}
&T^{00} = \frac{1}{8\pi}E^2 - \frac{2\pi}{k_0^2 c^2}\left(c^2\rho^2 + j^2\right); \quad T^{11} = -\frac{1}{8\pi}E^2 - \frac{2\pi}{k_0^2 c^2}\left(c^2\rho^2 + j^2\right) \\
&T^{22} = T^{33} = \frac{1}{8\pi}E^2 - \frac{2\pi}{k_0^2 c^2}\left(c^2\rho^2 - j^2\right); \quad T^{01} = -\frac{4\pi}{k_0^2 c^2} c\rho j
\end{aligned} \tag{13}
$$

To actually calculate a time average of the energy-momentum tensor we have to take the real parts of the physical values, multiply them and then take the time average. The result will be equal to ½ multiplied on the real part of the product of the first complex amplitude on the conjugate of the second complex amplitude. The result of calculation is (we do not keep the factor ½):

$$T^{33} = -(F_1^+)^2 \frac{2k_0^4}{\pi |D|^2}(k_0^2 + 2k^2); \quad T^{03} = -(F_1^+)^2 \frac{4k_0^2 k^2 k'^2}{\pi |D|^2}$$
$$T^{00} = (F_1^+)^2 \frac{2k_0^6}{\pi |D|^2} \cos(2k'(a-z)) \qquad (14)$$

The first two time averages of the tensor components appear to be independent of z. The energy density depends on z. All these tensor components are zero in the both vacuum regions. That means that all of them jump on the boundaries.

On the first boundary the jump of T^{33} is negative. It means that the first boundary will be pushed to the right. On the second boundary the jump will be positive and the same by its absolute value (because T^{33} is constant inside the second region). The second boundary will be pushed in the negative direction of z-axis with the same force – we have equilibrium – no "free" force.

On the first boundary the jump of T^{03} is negative. It means that the first boundary will be getting energy. On the second boundary the jump will be positive and the same by its absolute value (because T^{03} is constant inside the second region). The second boundary will be loosing the same amount of energy – no "free" energy.

Although, we have to notice that we encountered a new concept: the static boundary can gain or loose energy – is it possible? Also a little bit strange is the following: we know that the hard material body that is responsible for the existence of both boundaries can transmit the force from one boundary to another; what about energy? Is it also possible?

Being a conservative I would say: it is impossible! Instead: the energy is born from the free source on the second boundary and disappears in the "free" drain on the first boundary. On the first glance it looks that we can not benefit from that: there is a free source but it always complemented by a free drain of the equal amount that is separated in space from the source on a very small distance – the size of elementary particle.

Let us turn attention to T^{00} from (14). If $k'a < \pi/4$ then it is positive everywhere inside the material continuum. If $k'a > \pi/4$ then the energy density has positive and negative regions. It is possible to imagine that a small part of the energy from the regions where the density of energy is positive can be lost by the radiation of usual transverse E-M waves. In this case the drain of energy at the first boundary will get less energy than it was born at the second boundary (meaning that T^{03} will decrease by its absolute value). It is reasonable to suggest also that the flow of linear momentum T^{33} will decrease by the absolute value at the first boundary. That means that the push on the first boundary will decrease while the push on the second boundary will remain the same. It will explain gravitation! Notice also, that the gravitation and the free energy are two different sides of the very same phenomena.

Now I think in reverse: The actual existence of gravitation can be considered as a proof that if we have some positive energy density in space, then a small part of this energy can be lost in the form of usual E-M radiation.