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My Mistake: The Notion that There is no Electric Field inside a Capacitor is Wrong

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It is true that we can expand the plates of a plane capacitor as we like. But it is not true that the geometry of a capacitor will ever get the plane symmetry where all the physical quantities depend only on one coordinate. This was the reason for my mistake. The correct treatment of a plane capacitor is given below.

Suppose that a grounded source of alternating potential $\Phi_1(0,t)=F_1\exp(i\omega t)$ (with negligible internal resistance) is connected in series with a conducting left plate of a capacitor C which is located at z1=0, the conducting right plate is located at z2=a. The plates are round and we are going to use cylindrical coordinates z, ρ, φ which are numbered 1, 2, 3 correspondingly. The area of plates is S. The right plate is connected to the ground through a resistor R. The conventional solution of this problem is:

$$C=\frac{S}{4\pi a};\quad I=\frac{\Phi}{R}\frac{i\sigma_0}{1+i\sigma_0};\quad \sigma_0=\omega RC \tag{1}$$

We think that the vector potential does represent some physical reality that it is unique and satisfies the equations:

$$\Delta\Phi-\frac{1}{c^2}\ddot{\Phi}=-4\pi\rho;\quad \Delta\vec{A}-\frac{1}{c^2}\ddot{\vec{A}}=-\frac{4\pi}{c}j;\quad \frac{1}{c}\dot{\Phi}+div\vec{A}=0 \tag{2}$$

We can add to the vector potential a gradient of arbitrary (but good) function without changing the electromagnetic field. But the new potential will not have a physical meaning because the boundary conditions at infinity won't be satisfied (we suppose that the unique physical potential satisfies these conditions. The last equation in (2) is a Lorentz gauge for the potential (as seen from the realm of electromagnetic field). If we want to attribute a physical meaning to the potential we have to make it unique. Let us assume the Lorenz gauge as a law in the realm of potential and call it "potential conservation". After that, the uniqueness of potential still remains in question because of the freedom of jumping of a normal component. If so, the second and third equations in (2) lose their physical meaning on the boundary of disruption. Still, they have to be satisfied in the continuous regions. This freedom of jumping may disappear in the physical realm of potential if the potential is to be unique. As a working hypothesis we can assume that all the components of potential including their first derivatives have to be continuous on all the boundaries (including infinity).

Let us consider the solution of (2) in our symmetry:

$$\Phi(z,\rho,t) = J_0(k_2\rho)(+)e^{i\omega t}; \quad (\pm) \equiv F^+ e^{-ik_1 z} \pm F^- e^{ik_1 z}; \quad k_1^2 + k_2^2 = k^2; \quad k = \frac{\omega}{c}$$

$$A^2(z,\rho) = -\frac{ik}{k_2} J_1(k_2\rho)(+); \quad div\vec{A} = A^2_{1\rho} + \frac{1}{\rho} A^2; \quad H_3 = -\frac{kk_1}{k_2} J_1(k_2\rho)(-)$$

$$E_1 = ik_1 J_0(k_2\rho)(-); \quad E_2 = \frac{k_2^2 - k^2}{k_2} J_1(k_2\rho)(+) \tag{3}$$

$$I = \frac{i\omega}{4\pi} \int_0^{\rho_1} E_1 \rho d\rho d\varphi = -\frac{\omega k_1}{2k_2} \rho_1 J_1(k_2\rho_1)(-)$$

We denote the solutions: Φ(z,ρ,t) and A^2(z,ρ,t) in vacuum at 0<z<a. The vector potential has only radial component. Notice that a vector potential of a linear current has only z component (A^1). The solutions of the equations (2) in vacuum consist of waves going in different directions of the z axis.

In (2) we consider that every quantity has the factor exp(iωt) but we do not write it everywhere. J_0 and J_1 are cylindrical Bessell functions. The total current I is calculated as a total displacement current. In the expressions (3), for simplification, we can assume k_2=0:

$$\Phi(z,\rho,t) = (+)e^{i\omega t}; \quad k_2 = 0; \quad k_1 = k; \quad A^2(z,\rho) = -\frac{ik}{2}\rho(+)$$

$$H_3 = -\frac{k^2}{2}\rho(-); \quad E_1 = ik(-); \quad E_2 = -k^2\rho(+) \tag{4}$$

$$I = \frac{i\omega}{4\pi} \int_0^{\rho_1} E_1 \rho d\rho d\varphi = -\frac{\omega k}{4} \rho_1^2(-)$$

Suppose that the potential at the first plate appeared to be Φ(a,t)=F_2exp(iωt), and at the second Φ_2(a,t)=F_2exp(iωt). Then:

$$F^+ = \frac{F_2 - F_1 e^{ika}}{2\sin ka}; \quad F^- = \frac{-F_2 + F_1 e^{-ika}}{2\sin ka}; \quad ka \to small$$

$$(+) = F_1 \frac{\sin k(a-z)}{\sin ka} + F_2 \frac{\sin kz}{\sin ka} \approx F_1(1 - \frac{z}{a}) + F_2 \frac{z}{a} \tag{5}$$

$$(-) = -iF_1 \frac{\cos k(a-z)}{\sin ka} + iF_2 \frac{\cos kz}{\sin ka} \approx \frac{i}{ka}(F_2 - F_1)$$

Substituting in (4):

$$\Phi = F_1(1 - \frac{z}{a}) + F_2 \frac{z}{a}; \quad A^2 = -\frac{ik\rho}{2}\left[F_1(1 - \frac{z}{a}) + F_2 \frac{z}{a}\right]$$

$$H_3 = -\frac{ik\rho}{2a}(F_2 - F_1); \quad E_1 = \frac{1}{a}(F_1 - F_2); \quad I = \frac{i\omega\rho_1^2}{4a}(F_2 - F_1) \tag{6}$$

$$E_2 = -k^2\rho\left[F_1(1 - \frac{z}{a}) + F_2 \frac{z}{a}\right]; \quad I = i\omega C(F_2 - F_1)$$

It has to be noticed, that we consider a low frequency current so that the wave length is large compared to the sizes of the capacitor. That means that E_2 above can be neglected. From (6) we can conclude that $H_3=2I/c\rho_1$ which coincides with the usual magnetic field of a current formula.

I want to make a conclusion:

1. The usual one-dimensional treatment of AC circuits is a very rough approximation. The theory of a 3-dimensional field has to be used. Using the realm of potential gives, in my opinion, some simplification.
2. The Lorentz gauge should be upgraded to a physical law and the Coulomb gauge should be abandoned (relativistic invariance).
3. The potential has to satisfy the proper boundary conditions on all the disruption surfaces and at infinity. Consequently, it has to be unique.
4. It is possible that unique potential will be physically significant by itself in future (I mean besides the physical significance of the electromagnetic field that was incurred by the said potential).

References:

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