Classical Approach to the Theory of Elementary Particles

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Abstract: Presented here is an attempt to modify /extend classical electrodynamics (CED) in order to enable the classical approach (the approach based **only** on classical principles that were developed before the introduction of quantum theory) to the theory of elementary particles.

In physics, *dualism* refers to media with properties that can be associated with the mechanics of two different phenomena. Because these two phenomena's mechanics are mutually exclusive, both are needed in order to describe the possible behaviors.

(Retrieved from "http://en.wikipedia.org/wiki/Dualism"). Usually, dualism in physics is explained by the example: wave / particle. This started with a classical electromagnetic wave / quantum photon example for light and proceeded with a classical particle / quantum wave for massive particles. I want to perceive this dualism altogether as classical / quantum approach to the physical object. This sounds plausible (but we have to remember that these two approaches are "mutually exclusive" and never intermix them).

Taking the idea of classical approach to the elementary particles seriously, we can see that the existing classical electrodynamics (CED) has to be modified in 2 ways:
1. Many things were introduced in CED during the "quantum era" in order to narrow the gap between CED and Quantum theory and also in order to make CED a servant of Quantum theory. According to the classical principles that were established before the rise of Quantum theory, only the electromagnetic field and the field of current density were to represent the physical reality. The potential was not unique and was treated only as a mathematical tool. The Lagrangian was also thought of as the unique physical quantity that can not be expressed through potential. Here I mean the so called "interaction terms" in the Lagrangian that were introduced to ease the bridge between CED and the Quantum perturbation theory. The CED as we have it now is actually a part of Quantum theory. This should be undone (intelligently) to make CED a real alternative to Quantum theory (we expect that both theories will enjoy a partial experimental confirmation).
2. The CED (as it was in old times and is now) contains "singularities" as models for the elementary particles which deem to be "pointlike". I do not think that infinite quantities exist in physical reality. The true classical approach would be to replace these singularities by some extended field structures. Therefore CED has to be radically modified.

But the way to modify CED will not be the easy one as one may expect. So let us start with the simplest possible Lagrangian:

$$\Lambda = -\frac{1}{16\pi} g^{ab} g^{cd} F_{ac} F_{bd} - \frac{2\pi}{k_0^2} g^{ab} j_a j_b \tag{1}$$

where F_{ik} is the electromagnetic field, j_k is the field of current density, and k_0 will be the new constant of the theory. We suggest that the connection between the fields (also can be called “interaction”) is given by Maxwell's equations with currents (half of Maxwell's system):

$$j^i + \frac{c}{4\pi} F^{ik}{}_{|k} = 0, \quad div\vec{E} = \frac{4\pi}{c} j^0, \quad rot\vec{H} - \frac{1}{c}\dot{\vec{E}} = \frac{4\pi}{c}\vec{j} \tag{2}$$

which is given as a preliminary condition before any variation. The interaction condition (2) is an auxiliary condition with respect to the Lagrangian (1). The (2) also provides the conservation of charge: $j^a{}_{|a}=0$ (the antisymmetry of the electromagnetic field is also a preliminary condition). The other half of Maxwell's equations is:

$$F^{*ik}{}_{|k} = 0, \quad F^{*ik} \equiv \frac{1}{2} e^{iklm} F_{lm}, \quad div\vec{H} = 0, \quad rot\vec{E} + \frac{1}{c}\dot{\vec{H}} = 0 \tag{2a}$$

This will be the result of the variation procedure later on.

Before we start with the variation of fields, let us find the energy-momentum tensor that corresponds to the Lagrangian (1). The metric tensor in classical 4-space is g_{ik}=diag[1,-1,-1,-1] (we assume c=1). Let us consider an arbitrary variation of a metric tensor but on the condition that this variation does not introduce any curvature in space. This variation is:

$$\delta g_{ik} = \xi_{i|k} + \xi_{k|i} \tag{3}$$

where ξ^k is an arbitrary but small vector. One has to use the mathematical apparatus of General Relativity to check it with the variation (3) the Riemann curvature tensor remains zero in the first order. Assuming that the covariant components of the physical fields are kept constant (then the contravariant components will be varied as a result of the variation of the metric tensor, but we do not use them -- see (1)) we can calculate the variation of the action. The variation of the square root of the determinant of the metric tensor is: $\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}\, g_{ik} \delta g^{i|k}$ (this result can be found in textbooks on Theory of Field). The variation of the action becomes:

$$\delta S = \int \left\{ 2\frac{\partial\Lambda}{\partial g^{ik}} - \Lambda g_{ik} \right\} \xi^{i|k} \sqrt{-g}\, d\Omega = \int T_{ik} \xi^{i|k} \sqrt{-g}\, d\Omega$$
$$T_{ik} = -\frac{1}{4\pi} g^{ab} F_{ia} F_{kb} + \frac{1}{16\pi} F_{ab} F^{ab} g_{ik} - \frac{4\pi}{k_0^2} j_i j_k + \frac{2\pi}{k_0^2} j_a j^a g_{ik} \tag{4}$$

The integral in (4) can be transformed to the form $\int T^{ik}{}_{|k} \xi_i \sqrt{-g}\, d\Omega$ if we consider that the integral over some remote closed surface turns to zero due to the smallness of T_{ik} and the integral over 3-d volume at t1 and t2 turns to zero due to the assumption: ξ_i =0 at these times. Since ξ_i are arbitrary small functions (between t1 and t2), the requirement δS=0 yields:

$$T^{ia}{}_{|a} = 0 \qquad (5)$$

We have found the unique definition of the energy-momentum tensor (4). If we want action to be minimum with respect to the arbitrary variation of the metric tensor in flat space then (5) should be satisfied. Let us rewrite the energy-momentum tensor in 3-d form:

$$T^{00} = \frac{1}{8\pi}(E^2 + H^2) - \frac{2\pi}{k_0^2 c^2}[(j^0)^2 + (\vec{j})^2]$$

$$T^{11} = \frac{1}{8\pi}(E^2 + H^2 - 2E_1^2 - 2H_1^2) - \frac{2\pi}{k_0^2 c^2}[(j^0)^2 - (\vec{j})^2 + 2(j^1)^2]$$

$$T^{01} = \frac{1}{4\pi}(E_2 H_3 - E_3 H_2) - \frac{4\pi}{k_0^2 c^2} j^0 j^1$$

$$T^{12} = -\frac{1}{4\pi}(E_1 E_2 + H_1 H_2) - \frac{4\pi}{k_0^2 c^2} j^1 j^2 \qquad (4a)$$

Notice, that we did not used Maxwell's or any other field equations so far. Substituting (4) in (5) and using the Maxwell's equation (2) and antisymmetry of F_{ik}, we obtain:

$$j^a\left(\frac{k_0^2}{4\pi}F_{ai} + j_{a|i} - j_{i|a}\right) = 0, \quad j^0\left(\frac{k_0^2 c}{4\pi}\vec{E} + \nabla j^0 + \frac{1}{c}\dot{\vec{j}}\right) + \vec{j}\times\left(\frac{k_0^2 c}{4\pi}\vec{H} - rot\vec{j}\right) = 0 \qquad (6)$$

A new Dynamics

The equation (6) we can call a **Dynamics Equation.** It is nonlinear equation. It can be shown that any 3-d hypersurface that is everywhere tangent to the vector J^k is a **characteristic** of the equation (6). The equation (6) is automatically satisfied in vacuum ($J^k=0$). An another possibility ($J^k \neq 0$) will be the inside region of the elementary particle. The boundary between these regions will be a characteristic surface.

Inside the elementary particle the dynamics equation (6) describes, as we call it, **Material Continuum.** A Material Continuum can not be divided into a system of material points. The Relativistic (or Newton's) Dynamics Equation of CED, that describes behavior of the particle as a whole, completely disappears inside the elementary particle. There is no force, no velocity or acceleration inside the particle. A kinematic state of the Material Continuum is defined by the field of current density j^k. A world line of current j^k is not a world line of a material point. That allows us to deny any causal connection between the points on this line. In a consequence j^k can be space-like as well as time-like. That is in no contradiction with the fact that the boundary of the particle can not exceed the speed of light. If N_k is a normal to the boundary of a particle then $N^k j_k=0$ should be satisfied everywhere on this boundary. The scalar multiplication of N^k and (6) gives:

$$\frac{k_0^2}{4\pi}F_{ab}N^a j^b + j_{a|b}j^b N^a = \frac{1}{2}\left(\mu^2\right)_{|a}N^a, \quad \mu^2 \equiv j_a j^a$$

It is clearly seen that the left part of this equation is definite on the boundary of the particle. So should be and the right part. That means that on the boundary with vacuum the invariant density of charge should be zero.

By the equation (6) we have gotten something very important, but we are just on the beginning of a difficult and uncertain journey. Let us retrace some our steps.

1. **Ideal Particle (IP)** The first question was: is there a static solution to the system of equations (6) plus Maxwell's equations (2), (2a)? The solution was found:

$$j^0 = \frac{\alpha c k_0^2}{4\pi}R_0(z), \quad 0 \le z \le n\pi; \quad j^0 = 0, \quad n\pi \le z < \infty; \quad \vec{j} = 0 \tag{7}$$

$$E_r = \alpha k_0 R_1(z), \quad 0 \le z \le n\pi; \quad E_r = (-)^{n+1}\alpha k_0 \pi n / z^2, \quad n\pi \le z < \infty; \quad \vec{H} = 0$$

where $z = k_o r$, $R_o(z)=\sin(z)/z$, $R_1(z)=\sin(z)/z^2 - \cos(z)/z$ - the spherical Bessel functions, α is an arbitrary constant. The full charge of IP is:

$$e = 4\pi\int_0^{r_1} j^0 r^2 dr = (-)^{n+1}\frac{\alpha\pi}{k_0}n \tag{8}$$

To find the mass we should integrate T^{00} over the volume:

$$c^2 m = \int T^{00} r^2 \sin(\theta)\, dr d\theta\, d\varphi = \frac{8\pi^2}{k_0^2 c^2}\int [\frac{k_0^2 c^2}{16\pi^2}E^2 - (j^0)^2]r^2 dr =$$

$$= \frac{\alpha^2}{2k_0}\{\int_0^{z_1}[R_1^2(z) - R_0^2(z)]z^2 dz + (\pi n)^2\int_{z_1}^{\infty}\frac{dz}{z^2}\} = \frac{\pi\alpha^2}{2k_0}n \tag{9}$$

2. The second question was: is there a static solution that poses a spin? After a long series of unsuccessful attempts it was proven (see Appendix) the theorem that static spin is impossible with **any** energy-momentum tensor if its divergence vanishes everywhere and it is “good” on a remote sphere (static spin is possible in a static external magnetic field but the corresponding energy-momentum tensor is not “good” at infinity). The search for a dependent of time solution with spin gave no results.

After these unsuccessful attempts it was realized that the main result of the equation (6) is the boundary of the particle (characteristic of the corresponding PDE system). We did not expect it during the derivation of (6). We did not ask for “trouble”, so to speak, but we got it. The mere existence of the boundary of the elementary particle is a very important change in our mathematical apparatus and it requires all possible attention. Now, even before we can get the conservation equation (5) or perform any other variation, we have to take care of the boundary of the particle Σ. We have:

$$\delta S = -\int T_i^k \xi^i_{\ |k}\sqrt{-g}\,d\Omega = \int_\Sigma \left(T^k_{i\ out} - T^k_{i\ in}\right)\xi^i d\Sigma_k + \int_{in} T^k_{i\ |k}\xi^i\sqrt{-g}\,d\Omega + \int_{out} T^k_{i\ |k}\xi^i\sqrt{-g}\,d\Omega$$

Since ξ^k is arbitrary, we are coming to the conservation (5) inside and outside separately, and an additional requirement on the surface of disruption Σ:

$$T^{ia}N_a \quad continuous \tag{5a}$$

where N_k is a normal to the surface. Let us rewrite (5a) in 3-d form. Suppose the boundary of the particle is given in the form: f(t,y,z)-x=0. The covariant normal to this surface will be ($f_{,t}$ -1 $f_{,y}$ $f_{,z}$). The (5a) produces 4 conditions:

$$\left(E^2+H^2+\frac{16\pi^2}{k_0^2c^2}\mu^2\right)\dot{f}+2(\vec{E}\times\vec{H}\cdot\vec{n}) \quad -cont., \quad \mu^2\equiv(j^0)^2-(\vec{j})^2$$

$$2(\vec{E}\times\vec{H})\dot{f}+\left(E^2+H^2-\frac{16\pi^2}{k_0^2c^2}\mu^2\right)\vec{n}-2(\vec{E}\cdot\vec{n})\vec{E}-2(\vec{H}\cdot\vec{n})\vec{H} \quad -cont. \tag{5b}$$

Now we can vary the physical fields J_k and F_{ik} keeping in mind that the space is divided at least in two parts by some disruption surface. For that reason we have to apply a variation procedure separately for each region. We claim that the preliminary condition (2) holds in both these regions, but now we even not sure if it should hold on the boundary itself. In our system we have 10 unknown independent functions (4 functions in J_k and 6 functions in F_{ik}). These functions already have to satisfy to 8 equations (4 equations in (2) and 4 equations in (6)). We have only 2 degrees of freedom left. We can not variate neither J_k nor F_{ik} by the straight forward procedure. Let us employ here the Lagrange method of indefinite factors. Let us introduce a modified Lagrangian:

$$\Lambda'=\Lambda+A^a\left(j_a+\frac{1}{4\pi}F_{ab}{}^{|b}\right) \tag{10}$$

where A^k are 4 indefinite Lagrange factors. Now we have 2+4=6 degrees of freedom and we use them to vary F_{ik}. The variation of the field δF_{ik} we have to keep zero on the boundary since the derivatives of the field are present in (10). The Euler's equation gives:

$$F_{ik}=A_{k|i}-A_{i|k} \tag{11}$$

inside. The Maxwell's equation (2a) follows from (11). We are not able to very J_k since there are no degrees of freedom left, but we already have (6) as a dynamics equation.

If we do not keep the variation of field zero on the boundary then we will get, in addition to (11), the surface integral:

$$\frac{1}{4\pi}\oint\left(A^ig^{kl}-A^kg^{il}\right)_{in}\delta F_{ik}d\Sigma_l \tag{12}$$

which we want to get rid of (Σ is the boundary surface of the particle).

Let us consider the vacuum around the particle. If we use the same modified Lagrangian (10) in vacuum and vary the electromagnetic field in vacuum including the boundary of the particle, then in addition to the extension of (11) into the vacuum (still, it is not definite whether (11) holds on the boundary itself), we will get the surface integral:

$$-\frac{1}{4\pi}\oint\left(A^ig^{kl}-A^kg^{il}\right)_{out}\delta F_{ik}d\Sigma_l \tag{13}$$

Both integrals will annihilate if the tangent components of a vector potential A^k are continuous across the particle boundary. The component of potential that is perpendicular to the boundary of the particle can have a jump.

The potential Ak should be chosen so that it satisfies the preliminary condition (2). Using (2) and (11), and not assuming any gauge, we get:

$$A^{k|a}{}_{|a} - A^{a|k}{}_{|a} = 4\pi\, j^k \quad \textit{inside}, \quad A^{k|a}{}_{|a} - A^{a|k}{}_{|a} = 0 \quad \textit{vacuum} \tag{14}$$

In addition, as we learned above, the tangent components of potential have to be continuous across the boundary of the particle. The potential further reveals its strange nature. Notice, that the above derivation does not preclude a failure of Maxwell's equations (2) or (2a) on the boundary of the particle. We came at a very unusual arrangement:

1. The equations (6), (2), and (2a) define the solution for the electric, magnetic, and current density fields inside the particle.
2. The same equations define the solution for the electric and magnetic fields in vacuum.
3. There are no direct requirements on the components of electromagnetic field and 3 components of a current density on the boundary of the particle (only the component of current normal to the boundary should be zero). Instead, we have the 4 nonlinear algebraic conditions (5b) on electromagnetic field and currents, and the 3 direct conditions on the tangent components of potential (we do not care about the component of potential that is normal to the boundary).

That means that without the potential we can not solve the physical problem. Still, the potential is only a mathematical tool (it does not represent any physical reality in a point of space (contrary to the electromagnetic field and the field of current density)). This was the one of the classical principles before the quantum theory.

Comparison of the two approaches

Above we developed the two possible ways in which CED can be modified (extended) in order to approach the elementary particles from the **pure** classical point of view:

1. The "**boundary last**" approach. We requested that the "interaction" between the electromagnetic field and the field of current (expressed by the Maxwell's equation (2)) should be the preliminary condition true in all the space. We did not expect any disruption surface before the variation. We obtaind the system (2), (2a), (6) with the conserving energy-momentum tensor (4), (4a). This system is unlinear due to the equation (6). The characteristics of the system allow a surface of disruption in a solution. We obtained the static solution of this system (IP). The whole set of the solutions of the system was investigated purely, but I would like to say that having only Ideal Particle one can not explain much in the real world. The first earge was to do away with the quantization of the radius of IP. The corresponding particle was called IP+ and, shure enougth, IP+ violates the energy-momentum conservation. I am 90% sure that if we want to continue with the "boundary last" approach, we have to live with this unconservation. But we are not questioning the validity of the Maxwell's equations here.

It was mentioned above that IP+ violates the energy-momentum conservation. What does it mean? Let us consider two coordinate systems: K – the coordinate system in which the solution IP+ is given, and K' that moves with a speed V in x direction (we assume c=1). Using Lorentz transformation we obtain “image” of the IP+ solution in K'. If we calculate by integration a global (meaning not connected to any point of space) conserving linear momentum vector:

$$P^k = \int_{t=0} T^{k0} d_3V \tag{15}$$

in coordinates K, then we find only time component (energy) is different from zero: $P^0 \neq 0$. The corresponding integration in K' (over t'=0) will give another vector $Q^{k'}$ with $Q^{0'}$ and $Q^{1'}$ different from zero. Now by Lorentz transformation we can transform P^k from K to K' obtaining $P^{k'}$ (with 2 components different from zero). If the solution satisfies the conservation requirements, then we should get: $P^{k'}=Q^{k'}$ in K'. Let us see what we get for IP+.

$$P^0 = m = -\frac{\alpha^2}{2k_0} z_1^2 R_1(z_1)\cos(z_1) > 0, \quad P^1 = 0; \quad P^{0}{}' = \gamma m \approx m + \frac{m}{2}V^2, \quad P^{1}{}' = \gamma m V;$$

$$Q^{0}{}' = \gamma(m - m_1 V^2) \approx m + \frac{m - 2m_1}{2}V^2, \quad Q^{1}{}' = \gamma V(m - m_1), \quad m_1 = \frac{\alpha^2}{6k_0} z_1^3 R_0^2(z_1) \tag{16}$$

If z1=nπ then m1=0; m stays positive almost at all z1; the approximation is given for a small V. For IP+ it looks like the unconservation is playing games with the mass.

2. The "**boundary first**" approach. We state the existence of the boundary from the very beginning. We claim that the interaction (2) is valid inside and outside the particle separately. We obtained that (2), (2a), (6) are valid inside the particle, and (2), (2a) with a zero current are valid in vacuum. We do not question the energy-momentum consevation here. It brings us, in addition, the boundary conditions for the fields and currents (5a), (5b) which are unlinear in nature. In addition, we have the unknown in conventional CED requirement on the tangent components of potential on the particle boundary and on infinity. In this approach the Maxwell's equations can fail on the boundary of the particle (the electromagnetic field and currents can have a jump).

IP2: Let us obtain a simplest static solution in the "boundart first" approach with electric charge and electric field only. We have:

$$\begin{aligned}
&A^0 = \alpha\left(R_0(z) - R_0(z_1) + bz_1\right), \quad 0 \le z \le z_1; \quad j^0 = \frac{k_0^2 c}{4\pi}\alpha R_0(z); \\
&A^0 = \alpha b \frac{z_1^2}{z}, \quad z_1 \le z < \infty; \quad b \equiv \sqrt{R_0^2(z_1) + R_1^2(z_1)}; \\
&E_r = \alpha k_0 R_1(z), \quad 0 \le z \le z_1; \quad E_r = \alpha k_0 b \frac{z_1^2}{z^2}, \quad z_1 \le z < \infty; \\
&mc^2 = \frac{\alpha^2}{2k_0}\left(z_1^3 R_0^2(z_1) + z_1^3 R_1^2(z_1) - z_1^2 R_0(z_1) R_1(z_1)\right);
\end{aligned} \tag{17}$$

Now the position of the boundary z_1 is arbitrary. In general, the electric field has a jump on the boundary of IP2. Accordingly, the total charge has different "actual" and "effective" values (the "effective" charge is the one that corresponds to the vacuum field of the particle). If z1 is equal to nπ then the jump disappears and IP2 turns to IP. The IP2 fully satisfies to the energy-momentum conservation law (5), (5a), (5b). The potential A^0

is a tangent component, so, it should be continuous over the particle's boundary, and should have a reasonable value at infinity. If we want the integral (12) taken over the infinitely remote sphere to be zero then the potential has to decrease as r^{-2} or faster. For that reason we can always consider two particles with opposite charges that is very far one from another so that the interaction can be neglected, but still, they will make a proper dependence of the potential at infinity. The continuity of potential brings up a very interesting question: Two particles does not interact but the structure of the one can influence the structure of the other through the potential at infinity. Suppose our particle has a spin directed along z-axis. Then another particle of the similar structure but with untiparallel spin should exist somewhere in space (far, or close does not matter) in order to cancel the potential at infinity.

Magnificent Particle (MP)

Here we disscuss "**the boundary first**" approach. Above we saw the electric static solution (IP2) that violates Maxwell's equations on the boundary, but (and this is important) does not violate the principal of minimum action. We tried to obtain a magnetic static solution (static spin), but again, it was unsuccesslull. Now we are compelled that a spin should be looked for in a steady-state oscillating solution.

Warning: The MP is not an actual rigorous solution (because we mange to to satisfy the unlinear boundary condition (5b) only in average over the period of oscillation) but an interesting close shot on it.

Since the potential satisfies the linear equations (14) and the strait forward boundary conditions (tangent components should be continuous), it follows that the sum of two solutions of the equations (14) is also a solution (principal of superposition). The trouble arises when it comes to the unlinear boundary condition (5b) for the electromagnetic field and currents. We will disscuss a solution that contains 3 parts (we assume that the currents inside the particle are expressed through the potential $j^k = \frac{k_0^2 c}{4\pi} A^k$ with exception of the static electric part; also we assume the Lorentz gauge):

1. A static electric part:

$$A^0 = \alpha\left(R_0(k_0 r) - R_0(k_0 r_1) + \frac{e}{k_0 r_1} \right), \quad j^0 = \frac{k_0^2 c}{4\pi}\alpha R_0(k_0 r) \quad in; \quad A^0 = \frac{\alpha e}{k_0 r} \quad out \tag{18}$$

A^0 is a tangent component at the infinitely remote sphere. To cancel the potential there (and make it decrease as r^{-2}) we have to have another particle with $\alpha' e' = -\alpha e$ somewhere before the infinity. The corresponding electric field is:

$$E_r = \alpha k_0 R_1(k_0 r) \quad in; \quad E_r = \frac{\alpha e}{k_0 r^2} \quad out \tag{18a}$$

2. A static magnetic part:

$$A_\varphi = \beta R_1(k_0 r)\sin\theta \quad in; \quad A_\varphi = \beta R_1(k_0 r_1)\frac{r_1^2}{r^2}\sin\theta \quad out \tag{19}$$

We may do not need to cancel this at infinity because we already have a good decrease rate. The corresponding magnetic field is:

$$H_r = \beta\,\frac{2}{r}R_1(k_0 r)\cos\theta \quad in; \quad H_r = \beta\,\frac{2r_1^2}{r^3}R_0(k_0 r_1)\cos\theta \quad out$$

$$H_\theta = \beta\left(\frac{1}{r}R_1(k_0 r) - k_0 R_0(k_0 r)\right)\sin\theta \quad in; \quad H_\theta = \beta\,\frac{r_1^2}{r^3}R_0(k_0 r_1)\sin\theta \quad out \tag{19a}$$

3. And an oscillating part:

$$h_1 = \sin(\varphi - \omega t), \quad h_2 = \cos(\varphi - \omega t), \quad z = kr, \quad k^2 = k_0^2 + \omega^2$$

$$y_0(\omega r) = \delta_1 R_0(\omega r) + \delta_2 Q_0(\omega r), \quad y_1(\omega r) = \delta_1 R_1(\omega r) + \delta_2 Q_1(\omega r)$$

$$A^0 = \delta\, h_1 R_1(z)\sin\theta \quad in; \quad A^0 = h_1 y_1(\omega r)\sin\theta \quad out$$

$$A_r = \delta\, h_2\,\frac{\omega}{k}\left[(1-\gamma\,\frac{k_0^2}{\omega^2})\frac{2}{z}R_1(z) - R_0(z)\right]\sin\theta \quad in, \; A_r = h_2\left(\frac{2}{\omega r}y_1(\omega r) - y_0(\omega r)\right)\sin\theta \quad out$$

$$A_\theta = -\delta\, h_2\,\frac{\omega}{k}\left[(1-\gamma\,\frac{k_0^2}{\omega^2})\frac{1}{z}R_1(z) + \gamma\,\frac{k_0^2}{\omega^2}R_0(z)\right]\cos\theta \quad in, \quad A_\theta = -h_2\,\frac{1}{\omega r}y_1(\omega r)\cos\theta \quad out$$

$$A_\varphi = \delta\, h_1\,\frac{\omega}{k}\left[(1-\gamma\,\frac{k_0^2}{\omega^2})\frac{1}{z}R_1(z) + \gamma\,\frac{k_0^2}{\omega^2}R_0(z)\right] \quad in, \quad A_\varphi = h_1\,\frac{1}{\omega r}y_1(\omega r) \quad out \tag{20}$$

where $Q_0(x) = -\cos(x)/x$, $Q_1(x) = -\cos(x)/x^2 - \sin(x)/x$ are the second kind spherical Bessel functions. The potentials (20) satisfy (14) and the conservation condition: $A^k{}_{|k}=0$ inside. The (18) and (19) are written so that the boundary conditions at r1 already satisfied. To satisfy the boundary conditions for (20) we have to request:

$$\delta\, R_1(z_1) = y_1(\omega r_1); \quad \delta\,\frac{\omega}{k}\left[(1-\gamma\,\frac{k_0^2}{\omega^2})\frac{1}{z_1}\bar{R}_1 + \gamma\,\frac{k_0^2}{\omega^2}\bar{R}_0\right] = \frac{1}{\omega r_1}y_1(\omega r_1)$$

$$\gamma = \frac{\bar{R}_1}{(z_1\bar{R}_0 - \bar{R}_1)}; \quad \bar{R}_i \equiv R_i(z_1); \quad z_1 = k_0 r_1 \tag{21}$$

The constant γ is obtained as a result of the first two boundary conditions. On this conditions A^0, A_θ, and A_φ will be continuous (A_r is not continuous but we do not care about that). We have to cancel the oscillating potential at infinity because the spherical Bessel functions of both kinds decrease as r^{-1} (not good enough) at infinity. So we have to have another particle with $\delta_1' = -\delta_1$ and $\delta_2' = -\delta_2$ (also will be $\delta' = -\delta$) with the same orientation of z-axis. Note, that the functions h1(φ-ωt) and h2(φ-ωt) should be the same. If we change the sign of ω then it will be another solution. Let us calculate the electromagnetic field that corresponds to the oscillating potentials (20):

$$E_r = \delta \frac{k_0^2}{k} h_1 \left[(1+\gamma) \frac{2}{z} R_1(z) - R_0(z) \right] \sin\theta \quad in, \quad E_r = 0 \quad out$$

$$E_\theta = \delta \frac{k_0^2}{k} h_1 \left[\gamma R_0(z) - (1+\gamma) \frac{1}{z} R_1(z) \right] \cos\theta \quad in, \quad E_\theta = 0 \quad out$$

$$E_\varphi = \delta \frac{k_0^2}{k} h_2 \left[\gamma R_0(z) - (1+\gamma) \frac{1}{z} R_1(z) \right] \quad in, \quad E_\varphi = 0 \quad out$$

$$H_r = 0 \quad in, \quad H_r = 0 \quad out; \quad H_\theta = \delta\gamma \frac{k_0^2}{\omega} h_1 R_1(z) \quad in, \quad H_\theta = 0 \quad out$$

$$H_\varphi = \delta\gamma \frac{k_0^2}{\omega} h_2 R_1(z) \cos\theta \quad in, \quad H_\varphi = 0 \quad out \qquad (20a)$$

The amazing thing about this oscillating field is: The oscillating electromagnetic field inside the particle is present, but there are no oscillating electromagnetic fields in vacuum while the oscillating potential is present.

The expression for γ (see (21)) indicates that there exist the resonance frequencies that correspond to: z1=2.743707270, 6.116764264, 9.316615629,... (if k0 and r1 are given then only frequency defines z1) at which γ (and the whole solution) becomes infinite.

The investigation of the radial current $j_r = \frac{k_0^2 c}{4\pi} A_r$ (see (20)) on the boundary shows that it has minimums at z1 that satisfies the equation:

$$R_1(z_1) = \frac{2}{3} z_1 R_0(z_1) \qquad (22)$$

the solutions of which are: z1=2.460535572, 6.029292382, 9.261401926,... . If we take z1 satisfying (22) then from (21) we find γ=2 (the reason why we do not require j^r=0 on the particle boundary will be explained later).

We will seek the solution as a superposition of all the above 3 parts. But our boundary of the particle is too simplified (it is a static in time sphere of radius r1). We found that we are not able to satisfy the boundary conditions (5b) on the static sphere. We think that the real boundary has to be not a sphere (but close to it) and has to depend on time. Bound by the spherical coordinate system we were not able to find the real boundary (and so the actual solution). Still, we can hope that if we satisfy the boundary conditions (5b) on our sphere in average over a one period of the oscillation, then the real solution should exist somewhere not far from our approximation. These are also the reasons why we are seeking the radial current to be only a minimum on the sphere.

Let us take a closer look at the conditions (5b). We have the normal in spherical coordinates: $r_{,t}$=0, n_r=-1, n_θ=0, n_φ=0. The time component of the condition (5b) is satisfied because $(\vec{E} \times \vec{H} \cdot \vec{n}) = 0$. The phi component of (5b) is continuous in average. The time average of the theta component of (5b) (the difference inside minus vacuum) is proportional to:

$$\beta^2 R_0(k_0 r_1) R_1(k_0 r_1) \sin\theta \cos\theta$$

Let us take $k_0 r_1=\pi$ (which turns the above expression to zero) and z1=9.261401926. Then $k/k_0 \approx 2.948$, $\omega/k_0 \approx 2.773$. The k_0 still remains indefinite. At these values the radial current is very small compare to its neighboring terms. The time average of the radial component of the condition (5b) will be of the form: $A+B\sin^2\theta$. From A=0 we will get: $\alpha^2(1-e^2/k_0^2/r_1^2) \approx \delta^2 0.06188$, and from B=0 we will get: $\beta^2 \approx \delta^2 0.09054$. These conditions will define α and β (we still have the option to choose the sign of α and β as we like). After that we can calculate the effective charge and magnetic moment, and, after the integration over 3-space, the mass and the spin. All these values will be expressed through δ, k_0, and c_0. These 3 constants won't allow us to make the result looking as an electron or proton (because it requires 4 constants). On the top of that the values of the actual solution that satisfies (5b) can shift significantly. Still we only hope that this "actual" solution (MP) does exist, but this preliminary attempt looks promising. Since the solution has resonance frequencies we need to introduce a power dissipation to make a more realistic solution.

Conclusion:

I want to attract attention to the change of the variation procedure: instead of keeping the "interaction" term in the Lagrangian and variating potential (conventional procedure) we define interaction (2) as an auxiliary condition before the variation. Then, variating the electromagnetic field itself (not a potential), we use the Lagrange method of indefinite factors with the modified Lagrangian (10). The important result of this change of the variation procedure is the continuity of tangent components of the potential (and, consequently, definite value of them at infinity) is valid in the conventional electrodynamics also.

Appendix: The Spin of a Classical Physical System with Continuous (Including First Derivatives) and "Good" Behaving at Infinity Energy-Momentum Tensor in Statics is Zero

The flat space of real 4-dimensional independent variable x^k with Lorentz metric is assumed.

If we have a classical physical system described in that space and the energy-momentum tensor of that system obeys the conservation law, and if:
1. This energy-momentum tensor is continuous in 4-space and has continuous first derivatives everywhere in 4-space (it can have a brake of the second derivative on some closed 3-d surface Σ in that space).
2. This energy-momentum tensor is "good" at terrestrial infinity (so that the surface integral over a 3-sphere of a big radius can be neglected).
3. The energy-momentum tensor does not depend on time.
Then the angular momentum of this system (spin) is zero.

To prove this theorem let us first consider a conserving vector $j^k{}_{|k}=0$. Applying 4-d Gauss theorem separately to 4 volume inside and outside 3-d closed surface Σ we can prove that the integral:

$$e = \int_{t=0} j^0 d_3 V \qquad (1.a)$$

does not depend on time. The integration in (1) goes over the hypersurface $x^0=0$. In different coordinates it will be different hypersurface but the integral will have still the same value. It is a good example of a global thing. Let us elaborate on the meaning of a good global. Let us consider two rectilinear coordinate systems K' and K which are connected by the Lorentz transformation:

$$\begin{aligned} x^0 &= \gamma(x'^0 + Vx'^1), \quad x^1 = \gamma(x'^1 + Vx'^0) \\ x'^0 &= \gamma(x^0 - Vx^1), \quad x'^1 = \gamma(x^1 - Vx^0) \end{aligned} \qquad (2.a)$$

We have:

$$e = \int_{x'^0=0} j'^0 dx'^1 = \int_{x^0=0} j^0 dx^1 = \int_{x^0=0} \left[j'^0(x'^0, x'^1) + Vj'^1(x'^0, x'^1) \right] \gamma\, dx^1$$

Here the integration over dx^2 and dx^3 supposed to be performed but not indicated for simplicity. In the last integral we just expressed j^0 according to the transformation of a contravariant vector which transforms the same way as coordinates do. Instead of integrating over dx^1 in the last integral we can integrate over dx'^1 but we have to fulfill the condition $x^0=0$, or $x'^0=-Vx'^1$ and $dx'^1=\gamma dx^1$ since we keep $x^0=0$. We have:

$$e = \int_{x'^0=0} j'^0(0, x'^1) dx'^1 = \int_{x'^0=-Vx'^1} \left[j'^0(-Vx'^1, x'^1) + Vj'^1(-Vx'^1, x'^1) \right] dx'^1$$

Now suppose that our distribution of currents does not depend on time x'^0 in coordinates K'. Since V is arbitrary and the Lorentz transformation (boost) can involve any of the coordinates x^1, x^2, x^3 we should conclude that in this case the integral:

$$\int j'^k d_3 V' = 0 \quad if \quad k = 1, 2, 3 \qquad (3.a)$$

This integral differs from zero only if k=0.

Taking the energy-momentum tensor we have to be careful because the Gauss theorem only applies to a vector. To reduce a tensor to a vector we have to introduce a constant vector e_k (four linear independent constant vectors can be introduced in a flat space). Using the conserving vector $G^k=e_i T^{ik}$ and using the same logic we can prove that if T'^{ik} does not depend on time in coordinates K' then the integral:

$$\int T'^{ik} d_3 V' \neq 0 \quad if \quad i = k = 0 \quad only \qquad (4.a)$$

differs from zero only if i=k=0.

The same way we can prove that in statics the integral:

$$M'_{ik} = e_{iklm} \int x'^l T'^{mn} d_3 V' \neq 0 \quad if \quad n = 0 \quad only \qquad (5.a)$$

Now using (4) and (5) we can prove that in statics:

$$\int x'^i T'^{k0} d_3 V = 0 \quad if \quad i \neq 0, k \neq 0, i \neq k \qquad (6.a)$$

The consequence of (6) is that M'_{0k} (spin) equals to zero in statics.

The final conclusion is: An independent of time solution that has a conserving ($T^{ik}{}_{|k}=0$) everywhere energy-momentum tensor that is "good" at infinity can not have angular momentum different from zero.

References:

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