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THE BASIC PRINCIPLES OF CLASSICAL ELECTRODYNAMICS (CED)

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Abstract
The Classical Principles of Independent CED are reminded to the physical community. A new development: The Basic Principles of Extended CED are discussed.

Key words: classical theory of field, classical electrodynamics, variation principle, point and extended classical particles, basic classical principles

1. INTRODUCTION

The last textbook on Classical Electrodynamics by J. D. Jackson (third edition)[(1)] says: “Now (1990s) classical electrodynamics rests in a sector of the unified description of particles and interactions known as the standard model.” And further: “classical electrodynamics is a limit of quantum electrodynamics (for small momentum and energy transfers and large average numbers of virtual or real photons).”

Does this mean that Classical Principles also follow from Quantum Principles in the above-mentioned conditions?

We heard before, that Classical and Quantum Principles are not compatible and therefore it is forbidden to apply Classical Principles in the micro universe, which is a real estate of Quantum Principles**.** But if we take a look at the history of theoretical physics – we will find the attempts to apply the classical principles to the elementary particles. The first objective of the Classical Theory of Field was to represent everything including the elementary particles as a continuous field distribution of some physical density in the space of a 4-dimensional independent variable x^k. This task failed (both in classical and quantum theories) because the only solution that we were able to offer for the elementary particles was a singular solution. The self-energy of the charged elementary particle became infinite. A clear indication of this failure is: We use the global charge, the coordinates of inertia center, the linear momentum, and the spin of the particles in a primary description of the physical reality. In the Theory of Field these quantities have to be obtained by the integration of the fields over the space after the primary description of the physical reality is settled. The quote from A. Einstein[(2)] states:

> "Should it not be possible to explain the total inertia of the particles electromagnetically? It is clear that this problem could be worked out satisfactorily only if the particles could be interpreted as regular solutions of the electromagnetic partial differential equations. The Maxwell equations in their original form do not, however, allow such a description of particles, because their corresponding solutions contain a singularity. Theoretical physicists have tried for a long time, to reach the goal by a modification of Maxwell’s equations. These attempts have, however, not been crowned with success. Thus it happened that the goal of erecting a pure electromagnetic field theory of matter remained unattained for the time being, although in principle no objection could be raised against the possibility of reaching such a goal. The thing which deterred one in any further attempt in this direction was the lack of any systematic method leading to the solution. What appears certain to me, however, is that in the foundations of any consistent field theory, there shall not be in addition to the concept of field, any concept concerning particles. The whole theory must be based solely on partial differential equations and their singularity-free solutions."

In my opinion the statement that Classical and Quantum Principles are not compatible is true. That means (contrary to Jackson) that these principles are **independent**. And, if they are independent then Classical Principles have a legitimate right to try to succeed in the micro universe. Of course, we all know that Classical Principles failed in the micro world. But this is another story: today they fail, tomorrow they may succeed.

In an attempt to make order, let us introduce a new terminology. In the last century the basic principles of CED were adopted for use as a raw material in quantum theory. The corresponding CED we will call Quantum CED (or QCED). The Quantum Electrodynamics (QED) can be obtained naturally from QCED as well as QCED can be obtained as a limit of QED.

The basic principles of really Independent CED (or ICED) look very different from the ones of QCED, and, I am afraid, that physicists are about to forget them. In this article I will try to remind the reader of The Basic Principles of ICED and introduce a new development: Extended CED (or ECED).

The very important principal of ICED is the **uniqueness of classical representation principal, which** works in concordance with A. Einstein's general covariance postulate. In ICED the physical reality is represented **uniquely** by the mathematical identities—numbers (we relate scalars, vectors, tensors to the category of numbers or identities) which we call **physical values**. The Physical Laws are expressed by the mathematical relations between the physical values. The Einstein's **general covariance** postulate(3) applies to the physical values as well as to the physical laws. It reads:

> "**We shall be true to the principle of relativity in its broadest sense if we give such a form to the laws that they are valid in every such 4-dimentional system of coordinates, that is, if the equations expressing the laws are co-variant with respect to arbitrary transformations.**"

The physical values form the exact boundary between physical reality and physical theory and, at the same time, they form the boundary between physics (represented by the physical values) and mathematical apparatus that is used to evaluate the physical values.

In consequence of the uniqueness we have to assume that only the second rank antisymmetric tensor F^{ik} can represent the physical reality called electromagnetic field. The electromagnetic potential A^k is part of a mathematical apparatus that we use to describe the electromagnetic field; it is not a physical value because it is not unique and because the electromagnetic field is already represented by the tensor F^{ik} which is the physical value.

2. BASIC EQUATIONS

The basic equations of QCED as well as conventional ICED are Maxwell's equations in vacuum:

$$F^{*ik}{}_{|k} = 0, \quad F^{*ik} \equiv \frac{1}{2} e^{iklm} F_{lm}, \quad div\vec{H} = 0, \quad rot\vec{E} + \frac{1}{c}\dot{\vec{H}} = 0 \quad (1)$$

$$j^i + \frac{c}{4\pi} F^{ik}{}_{|k} = 0, \quad div\vec{E} = \frac{4\pi}{c} j^0, \quad rot\vec{H} - \frac{1}{c}\dot{\vec{E}} = \frac{4\pi}{c}\vec{j} \quad (2)$$

and Relativistic Dynamics equation for an elementary particle:

$$m\frac{du^i}{ds} = \frac{e}{c} F^{ik} u_k, \quad m\frac{d}{dt}\left(\frac{\vec{v}}{\sqrt{1 - v^2/c^2}}\right) = e\vec{E} + \frac{e}{c}\vec{v} \times \vec{H} \quad (3)$$

where F^{*ik} is a "dual" (to F^{ik}) tensor, e^{iklm} is a fully antisymmetric tensor; the vertical line | denotes a covariant derivative with respect to the coordinate the index of which follows the line; any repeated indexes means summation. These equations are given in 4-covariant and 3-covariant forms (any equation given in 4-covariant form can be written in 3-covariant form but not in reverse). The current density j^i in (2) is a statistical value referring to the number of elementary charges in a unit of volume.

We have the problems (confusions) from the very beginning:

1. The equations (1) and (2) are field equations, while the equation (3) is a global equation (whether it applies to macroscopic body or to single elementary particle).

2. If we apply these equations to macroscopic bodies and statistical currents the whole theory becomes statistical (not classical). Further down we apply these equations to a finite number of elementary particles (one for simplicity) to be sure that we have a classical theory.

3. The claim that CED applies only to macroscopic bodies is wrong because the equation (3) holds true for a single elementary particle.

3. VARIATION PRINCIPLE

The variation principle is the very basis of any CED. It states that the variation of the action integral should be zero:

$$S = \int \Lambda \sqrt{-g}\, d\Omega, \quad d\Omega = dx^0 dx^1 dx^2 dx^3, \quad \delta\, S = 0 \quad (4)$$

Where x^0=ct, g is a determinant of a metric tensor, δ is a Lagrange density which can depend on the components of a metric tensor g^{ik}, on coordinates, on field functions A_i , F_{ik} , their derivatives $A_{i|k}$, $F_{ik|l}$, and trajectories of elementary particles: $x^i=x^i(t)$.

Although the minimum action statement is very clear and unique, we still have a large room of uncertainty depending on the choice of Lagrange density, field functions, and the choice of the variation procedure itself.

In **QCED** we take $F_{ik}=A_{k|i}-A_{i|k}$ before the variation. This way we satisfy the first half of Maxwell's equations (1). The Lagrange density of QCED is:

$$\begin{gathered} \Lambda = \Lambda_f + \Lambda_i + \Lambda_m, \;\; \delta_0 \equiv \left(-g\right)^{-1/2} \delta(x^1 - x^1(t))\delta(x^2 - x^2(t))\delta(x^3 - x^3(t)) \\ \Lambda_f = -\frac{1}{16c\pi} F_{ab}F^{ab}; \quad \Lambda_i = -\frac{e}{c^2} A_a \dot{x}^a \delta_0; \quad \Lambda_m = m\sqrt{\dot{x}_a \dot{x}^a}\,\delta_0 \end{gathered} \quad (5)$$

Where a dot denotes time derivative, δ is a delta function, m and e are the mass and charge of the elementary particle (for simplification consider only one elementary particle is present in the physical system). In (5) we use a field description of an elementary particle which is equivalent to the trajectory description. The quantity $e\dot{x}^i \delta_0 = j^i$ is a density of current.

As a result of variation of potential δA_i and the use of the Euler's equation:

$$\left(\frac{\partial \Lambda}{\partial q_{|k}}\right)_{|k} = \frac{\partial \Lambda}{\partial q}$$

(where q is any quantity that we very and upon which the Lagrangian depends)

we obtain the second part (2) of Maxwell's equations.

The variation of trajectory $\delta x^i(t)$ affects only $?_i$ and $?_m$. The corresponding part of action should be integrated over dx, dy, dz . The use of the Euler equation gives us the Relativistic Dynamics equation (3).

When it comes to the energy-momentum tensor QCED prefers to leave the matter uncertain. QCED claims that there are plenty of conserved quantities around. We just postulate the one we like, prove its conservation (given the Maxwell's equations (1), (2) and Dynamics equation (3) are satisfied), and go from there.

From the point of view of **ICED** the two things are wrong:

1. The Lagrange density (5) is not unique due to the "interaction term" which contains potential. Therefore it does not have a physical meaning.

2. We can not vary potential, which is not a physical value. We can vary electromagnetic field and trajectory.

The Lagrangian in ICED is:

$$\Lambda = -\frac{1}{16\pi} g^{ac} g^{bd} F_{ab} F_{cd} + m\delta_0 \sqrt{g^{ab} \dot{x}_a \dot{x}_b} \quad (6)$$

This Lagrange density is unique because it does not depend on the potential and, therefore, can be thought of as a direct representation of some physical reality.
The metric tensor in the classical 4-space (which is definitely flat) is g_{ik}=diag[1,-1,-1,-1]. Let us consider an arbitrary variation of the metric tensor but on the condition that this variation does not introduce any curvature in space. This variation is:

$$\delta g_{ik} = \xi_{i|k} + \xi_{k|i} \quad (7)$$

where ζ^k is an arbitrary but small vector. One has to use the mathematical apparatus of General Relativity to check that with the variation (7) the Riemann curvature tensor remains zero in the first order. Taking this variation we can calculate the variation of action. The variation of the square root of the determinant of the metric tensor is: $\delta\sqrt{-g} = -\sqrt{-g}\,\xi^{a}{}_{|a}$. The variation of action becomes:

$$\delta S = -\int T_{ik} \xi^{i|k} \sqrt{-g}\, d\Omega, \quad T_{ik} = 2\frac{\partial \Lambda}{\partial g^{ik}} - g_{ik}\Lambda \quad (8)$$

Evaluating Tik we get:

$$\begin{aligned} T_{ik} &= C_{ik} + 2m\frac{\partial \delta_0}{\partial g^{ik}}\sqrt{\dot{x}_a \dot{x}^a} + m\delta_0 \frac{\dot{x}_i \dot{x}_k}{\sqrt{\dot{x}_a \dot{x}^a}} - g_{ik} m\delta_0 \sqrt{\dot{x}_a \dot{x}^a} \\ C_{ik} &\equiv -\frac{1}{4\pi} g^{ab} F_{ia} F_{kb} + \frac{1}{16\pi} F_{ab} F^{ab} g_{ik} \end{aligned} \quad (9)$$

Recalling how $?_0$ depends on g_{ik} from (5), we see that the second and the fourth terms in Tik will annihilate and we obtain:

$$T_{ik} = C_{ik} + m\delta_0 \sqrt{\dot{x}_a \dot{x}^a}\, u_i u_k \quad (10)$$

which is the energy-momentum tensor of a point particle. The integral in (8) can be transformed to the form $\int T^{ik}{}_{|k} \xi_i \sqrt{-g}\, d\Omega$. Since δ_i are arbitrary small functions the requirement δS=0 yields:

$$T^{ik}{}_{|k} = 0 \quad (11)$$

We have found the unique definition of the energy-momentum tensor (10). If we want action to be minimum with respect to the arbitrary variation of the metric tensor in flat space then (11) should be satisfied. Contrary to QCED the conservation (11) is not a consequence – it is the requirement.

Let us now variate the field δF^{ik} and the trajectory $\delta x^i(t)$. Let us take the second part (2) of Maxwell's equations as a prerequisite before the variation. We will use here the method of indefinite Lagrange factors. For that purpose we will seek the minimum of action with a modified Lagrange density:

$$\Lambda' = -\frac{1}{16\pi} F_{ab} F^{ab} + m\delta_0 \sqrt{\dot{x}_a \dot{x}^a} + A^a \left(j_a + \frac{c}{4\pi} F_{ab}{}^{|b} \right) \quad (6a)$$

where A_a are 4 indefinite Lagrange factors that are to be chosen after the variation so that (2) will be satisfied.

By varying F^{ab} and using the Euler equation we get:

$$F_{ik} = A_{k|i} - A_{i|k} \quad (12)$$

As a result we have the first part (1) of Maxwell's equations satisfied. The Lagrange factors A_k clearly correspond to the usual vector potential and should be chosen to satisfy (2):

$$A^a{}_{|ai} - A_{i|ab} g^{ab} + \frac{4\pi}{c} j_i = 0 \quad (13)$$

Since $g^{ab} A_{i|ab} = -$?A_i where ? is a D'Alambert's operator, we are getting the familiar equation (the Lorentz gauge $A^k{}_{|k}=0$ is assumed):

$$\Box A_i = -\frac{4\pi}{c} j_i \quad (13a)$$

We have to come to the conclusion that the 4-potential A_k does not represent directly any physical reality, -- it is just our mathematical tool.

To prepare for the varying of trajectory $\delta x^i(t)$ let us perform the integration in action integral with modified Lagrangian (6a) over x,y,z. The dependent from trajectory terms will give:

$$S \sim \int \left\{ m\sqrt{\dot{x}_a \dot{x}^a} + eA_a \dot{x}^a \right\} dt$$

Variating the trajectory and using Euler equation we will obtain the Relativistic Dynamics equation (3).

Now let us find out what is necessary for the conservation (11) to be satisfied. Using Maxwell's equations we can evaluate $C_i^k{}_{|k} = -\frac{1}{c} F_{ia} j^a, \quad \left(u^k \delta_0 \sqrt{\dot{x}_a \dot{x}^a} \right)_{|k} = \frac{1}{e} j^k{}_{|k} = 0$. Using this we can prove that the requirement of conservation (11) also leads us to the Relativistic Dynamics equation (3), the same result as the variation of trajectory. It looks a little bit strange: we have more symmetry than we expected (two different ways give the same result).

We successfully finished describing the basic principles of ICED. The developed mathematics will allow us to extend CED.

In ECED the Lagrange density is:

$$\Lambda = \Lambda_f + \Lambda_m \quad \Lambda_m = -\frac{2\pi}{k_0^2 c^2} j_a j^a + p \quad (14)$$

where k_0 is some new universal constant that is inverse to the fundamental size of the classical particles, p is some pressure.

The second "material" term in ? does not contain any delta functions. Now we consider the current density j^i is a real continuous physical field – not a statistical value. The space is now divided into the regions, which we call the vacuum region and the inside region of the elementary particle. Between these regions there exist boundaries. As we will see later, these boundaries are the characteristic surfaces of the dynamics equations. They are called Free Boundaries. This material term allows us to describe "extended" particles and put an end to the so-called "Point Charge Era" of the 20th century. I am very positive about the usefulness of this term.

The third term is pressure that is not directly connected to the electromagnetic field or to the field of current density. It may indicate a presence of yet another physical reality that can be characterized only by the pressure (Ether?). We are not sure whether it is plausible to consider this term or not. But we can put it to zero at any time.

Inside region of the particle. We use here the same procedure as in the case of ICED. We keep the Maxwell's equation (2) as a preliminary condition and use the modified Lagrangian (6a) where ?$_m$ is taken from (14). We are leaving the discussion of the conditions on the boundary of the particle Σ for the special section. Varying the field δF^{ik} and applying the Euler equation we are getting the equation (12). The variation of the currents δj^i, according to Euler's equation, gives:

$$\frac{k_0^2 c}{4\pi} A_i = j_i, \quad or \quad \frac{k_0^2 c}{4\pi} F_{ik} + j_{i|k} - j_{k|i} = 0 \quad (15)$$

Here A_i becomes unique in the first equation; the second equation is gauge invariant. Using (13) we obtain:

$$\Box j^i + k_0^2 j^i = 0 \quad (16)$$

(Notice, that this equation contains + sign while the Klein Gordon equation contains -). According to the Euler's equation the variation of the pressure gives p=0.

Vacuum. In vacuum we put j^k=0 in the equations (2) and (14). We do not vary the currents because they are zero by assumption. By varying the field we get the equation (12) and by varying the pressure we get p=0.

Boundary of the Particle Σ. The modified Lagrange density does not depend on the derivative $j_{i|k}$. This allows us to keep the variation δj^k arbitrary on the boundary as well as inside the particle. If we keep the variation δF^{ik} arbitrary everywhere including the boundary of the particle, then the variation of the field inside will produce the surface integral

$$\int_\Sigma \left(\frac{\partial \Lambda'}{\partial F^{ik}{}_{|l}}\right)_{in} \delta F^{ik} d\Sigma_l$$

The variation in vacuum will produce the same integral with the minus sign in front of it and "in" replaced by "out". We have to get rid of these integrals in order to have the variation of the action zero. One of the ways is to keep the variation of the field zero on Σ. In this case the Maxwell's equations can fail on the particle's boundary. There is the other way: the "in" and "out" integrals will annihilate each other if the vector potential A^k is continuous across the particle boundary (can be checked out by evaluating the bracketed term under the integration). Only in this case the Maxwell's equations will be satisfied on the particle's boundary ∑.

In this variation procedure we do not have a trajectory to vary but, instead, we have the field of current density j^k. That leads us to think that (15) is an equivalent of a dynamics equation. But the Eq. (15) holds only inside the elementary particle. Here we are definitely missing something.

Energy-momentum tensor. By the variation of the metric tensor (7) we will get the energy-momentum tensor:

$$T_i^{\ k} = C_i^{\ k} - \frac{4\pi}{k_0^2 c^2} j_i j^k + \frac{2\pi}{k_0^2 c^2} j_a j^a \delta_i^{\ k} - p\delta_i^{\ k} \quad (17)$$

But now, before we can get the conservation equation (11), we have to take care of the boundary of the particle ∑. We have:

$$c\delta S = -\int T_i^k \xi^i{}_{|k} \sqrt{-g}\, d\Omega = \int_{\Sigma} \left(T_i^k{}_{out} - T_i^k{}_{in}\right)\xi^i d\Sigma_k + \int T_i^k{}_{|k} \xi^i \sqrt{-g}\, d\Omega$$

Since ζ^k is arbitrary we are coming to the conservation (11) and an additional requirement on the surface of disruption ∑:

$$T_i^k N_k \quad \textit{continuous} \quad (11a)$$

(where N_k is a normal to the surface) should be continuous across the surface. The Eq. (11) gives us a real dynamics equation:

$$\left(\frac{k_0^2 c}{4\pi} F_{ik} + j_{i|k} - j_{k|i}\right) j^k + \frac{k_0^2 c^2}{4\pi} p_{|i} = 0 \quad (18)$$

This equation is not a linear equation. The Eq. (18) supposed to be fulfilled everywhere including the boundary of the particle. In (18) we have gotten something more in comparison to (15). Now everything gets in its right place: we do not have "too much symmetry" as in the ICED case.

The equation (18) naturally introduces the boundary of the particle. Every 3-d surface to which the vector j^k is tangent in any point is a characteristic of the equation (18). Indeed: suppose that N^k is a normal to some 3-d surface and $N^k j_k=0$ everywhere. The scalar multiplication of N^k and (18) gives:

$$\frac{k_0^2 c}{4\pi} F_{ab} N^a j^b - j_a N^a{}_{|b} j^b = \frac{1}{2}\left(\mu^2 - \frac{k_0^2 c^2}{2\pi} p\right)_{|a} N^a,\ \mu^2 \equiv j_a j^a \quad (19)$$

where we used: $0=j_{a|b}N^a+j_a N^a{}_{|b}$. The left part of this equation is definite on the boundary of the particle. So should be the right part. This is a requirement on the boundary that originates from the dynamics equation. This requirement is in full agreement with (11a).

In our PDE system we have 4 unknown functions j^k and only one derivative across the surface is definite (it can be proven that there are no other definite derivatives across the boundary). That means that this surface is a characteristic of the corresponding PDE system. It should be mentioned that these characteristics form not only the boundary between elementary particle and vacuum but also can exist inside the elementary particle.

The dynamics equation (18) describes, as we call it, **Dynamics of Material Continuum.** A material

Continuum exists inside the elementary particles and can not be divided into a system of material points. The Dynamics Equation (3), that describes behavior of the particle as a whole, completely disappears inside the particle. There is no force, no velocity or acceleration inside the particle. A kinematical state of the material continuum is defined by the field of current density j^k. A world line of current j^k is not a world line of a material point. That allows us to deny any causal connection between the points on this line. In consequence the: j^k can be space-like as well as time-like. That is in no contradiction with the fact that the boundary of the particle can not exceed the speed of light.

In ECED our PDE system written in 4-covariant form consists of Maxwell's equations (1) and (2) and dynamics equation (18). To discuss the initial boundary problem let us write it in 3-d form:

$$div\vec{H} = 0, \quad div\vec{E} = \frac{4\pi}{c} j^0 \quad (20)$$

$$\frac{1}{c}\dot{\vec{E}} = rot\vec{H} - \frac{4\pi}{c}\vec{j}, \quad \frac{1}{c}\dot{\vec{H}} = -rot\vec{E}, \quad \frac{1}{c}\dot{j}^0 = -div\vec{j} \quad (21)$$

$$\frac{1}{c}j^0\dot{\vec{j}} = -j^0\left(\frac{k_0^2 c}{4\pi}\vec{E} + \nabla j^0\right) - \vec{j}\times\left(\frac{k_0^2 c}{4\pi}\vec{H} - rot\vec{j}\right) + \frac{k_0^2 c^2}{4\pi}\nabla p \quad (22)$$

$$\frac{1}{c}j^0\dot{p} = -\vec{j}\cdot\nabla p, \quad j^0 N^0 = \vec{j}\cdot\vec{N} \quad (23)$$

We have 11 unknown functions. Given initial data at t=0 we have to fulfill the conditions on the initial data (20) and make sure that $?^2$=0 on the boundary of the particle. The rest of the equations will define the time derivatives for all unknown functions and N^0 which defines where will be the boundary of the particle in the immediate future.

The development of ECED. As it follows from above, the ECED is an alternative to the quantum theory in an attempt to explain the elementary particles. How far did we manage to go in this direction?

In reference [4] we presented Ideal Particle (IP)– the perfect static solution in vacuum that has charge and mass but does not have spin. We should add to that another static solution in an external constant electric field called Dipole Particle (DP). Also we should add a solution in a static magnetic field called Static Spin Particle (SSP). This solution does not contradict the static spin zero theorem due to the fact that a static external magnetic field remains constant at infinity. These solutions are not published but they can be obtained without any complications. Nowhere do these static solutions violate the system.

As is prescribed by (19) any particle on the boundary with vacuum has to have $?^2$=0 (if p=0). It was shown in reference [4] that if we brake this requirement we can have a static solution with spin and can describe any particle that has a given mass, charge, spin, and magnetic moment, assuming just a spherical shape of the particles. In reference [5] we found two solutions for cylindrical particles that propagate with the speed of light. They are called Half Particle and Particle of Light. These solutions also violate (19) on the cylindrical surface. These developments force us to think that we are unable to find a right solution due to complexity (or we missing something in our basic equations—this is why I retain pressure p in the above formalism). The main difficulties we are having are, somehow, connected to the disruption surfaces. We have to learn how to obtain solutions where the boundaries of the particles are moving with acceleration.

4. A COUPLE OF IMPORTANT THEOREMS

The Correlated Jump Theorem.

It was shown in reference [4] that Maxwell's equations allow a discontinuity of the electromagnetic field on the characteristic surfaces of Maxwell's equations (these are the wave fronts). If on one side of the characteristic the vectors of electric and magnetic fields are:

a). tangent to the characteristic, b) perpendicular to each other, and c) equal in magnitude, then on the other side of the characteristic the electromagnetic field can be zero.

This theorem is useful in handling the particles that move with the speed of light.

The Static Spin Zero Theorem.

If we have a classical physical system and the energy-momentum tensor of that system obeys the conservation law, and if:

a) This energy-momentum tensor is continuous in 4-space and has continuous first derivatives everywhere in 4-space except some closed 3-d surface $\sum$ in that space.

b) This energy-momentum tensor is "good" at terrestrial infinity (so that the surface integral over a 3-sphere of a big radius can be neglected).

c) The energy-momentum tensor does not depend on time.

Then the angular momentum of this system (spin) is zero.

To prove this theorem let us consider at first a conserving vector $j^k_{|k}=0$. Applying 4-d Gauss theorem separately to a 4-d volume inside and outside 3-d closed surface $\sum$ we can prove that the integral:

$$e = \int j^0 d_3V \quad (24)$$

does not depend on time. The integration in (24) goes over the hypersurface $x^0=0$. In different coordinates it will be different hypersurfaces but the integral will be still the same due to conservation and Gauss theorem. It is a good example of a global thing. Let us elaborate on the meaning of a good global. Let us consider two rectilinear coordinate systems K' and K which are connected by the Lorentz transformation:

$$\begin{aligned} x^0 &= \gamma(x'^0+Vx'^1), \quad x^1 = \gamma(x'^1+Vx'^0) \\ x'^0 &= \gamma(x^0-Vx^1), \quad x'^1 = \gamma(x^1-Vx^0) \end{aligned} \quad (25)$$

We have:

$$e = \int_{x'^0=0} j'^0\, dx'^1 = \int_{x^0=0} j^0 dx^1 = \int_{x^0=0} \left[j'^0(x'^0, x'^1) + Vj'^1(x'^0, x'^1) \right] \gamma\, dx^1$$

Here the integration over dx^2 and dx^3 supposed to be performed but not indicated for simplicity. In the last integral we just expressed j^0 according to the transformation of a vector. Instead of integrating over dx^1 in the last integral we can integrate over dx'^1 but we have to fulfill the condition $x^0=0$, or $x'^0=-Vx'^1$ and $dx'^1=\gamma dx^1$ since we keep $x^0=0$. We have:

$$e = \int_{x'^0=0} j'^0(0, x'^1) dx'^1 = \int_{x'^0=-Vx'^1} \left[j'^0(-Vx'^1, x'^1) + Vj'^1(-Vx'^1, x'^1) \right] dx'^1$$

This is a remarkable result. Suppose our distribution of currents does not depend on time x'^0 in coordinates K'. Since V is arbitrary and the Lorentz transformation can involve any of the coordinates x^1,x^2,x^3 we should conclude that in this case the integral:

$$\int j'^{k}\, d_3V' = 0 \quad if \quad k = 1,2,3 \quad (26)$$

This integral differs from zero only if k=0.

Taking the energy-momentum tensor and using the same logic we can prove that if T'^{ik} does not depend on time in coordinates K' (in statics) then the integral:

$$\int T'^{ik}\, d_3V' \neq 0 \quad if \quad i = k = 0 \quad only \quad (27)$$

differs from zero only if i=k=0.

The same way we can prove that in statics the integral:

$$M'_{ik} = e_{iklm} \int x'^{l}\, T'^{mn}\, d_3V' \neq 0 \quad if \quad n = 0 \quad only \quad (28)$$

Now, using (27) and (28), we can prove that in statics:

$$\int x'^{i}\, T'^{k0}\, d_3V = 0 \quad if \quad i \neq 0, k \neq 0, i \neq k \quad (29)$$

The consequence of (29) is that M'_{0k} (spin) equals to zero in statics.

The final conclusion: A solution independent of time can not have angular momentum. But remember that all these conclusions are true only if: T^{ik} continuous and the conservation (12) holds everywhere inside some distant sphere that encloses the system under consideration; also if the integral of T^{ik} over this sphere can be neglected.

5. DISCUSSION

The QCED and ICED are based upon the PDE system (1), (2), (3). The ECED is based upon the PDE system (1), (2), (18). From the mathematical theory of PDE systems one knows that the characteristics of a PDE system play a profound role in finding the solutions of that PDE system. The characteristics allow the disruptive solutions to be the legitimate solutions of the corresponding PDE system. That means that the Lagrangian in general depends upon the field functions that have some kind of trouble on some 3-d surface ∑. This should be the standard approach of any variation procedure. In QCED and ICED it will be the wave front. In ECED it will be both, the wave front and the boundary of the particle. It is still not clear how to deal with these disruptions. There are some discrepancies between the results of the variation of the field functions and the variation of the metric. For example: the variation of the current gives the equation (15), which does not contradict to the equation (18) but “narrows” its scope. The same can be said about the pressure p. The solutions that were obtained so far satisfy to both equations in the inside and outside regions.

When we write the PDE system we include (18) but ignore (15) because the last is local. Actually, we are asking here the question: which variation procedure is more important? Can the local variation procedures say something more about the conditions on the disruption surfaces compare to what follows from the general PDE system?

If we try to build another conserving quantity in ECED the way it is usually done in QCED then we have to use the modified Lagrangian. This Lagrangian does not depend explicitly on coordinates. This fact allows us to construct a conserving quantity that has a zero divergence everywhere except the particle’s boundary where it will be indefinite. The famous statement that the conservation of linear momentum is a consequence of the homogeneity of the space turns out not to be true in ECED.

Describing the basic principles of ICED we showed that:

1. All the physical quantities (Lagrange density and energy-momentum tensor) can have a unique definition.

2. The so-called “interaction” term $A_k j^k$ is not a necessary part in a Lagrange density.

3. The actual meaning of the electromagnetic potential A_k is: it is an indefinite Lagrange factor used in the minimum action procedure. It is identified as a mathematical tool.

4. The conservation of the unique energy-momentum tensor is not a consequence – it is a requirement that follows from the minimum action with respect to the variation of a metric tensor in a flat space.

References:

1. J. D. Jackson, Classical Electrodynamics, third edition, Wiley, New York, (1999).

2. A. Einstein, *Out of My Later Years* (1950), p. 74.

3. A. Einstein, *The Meaning of Relativity*, (1921), p. 61.

4. Y. Keilman, Phys. Essays **11**, 34, (1998).

5. Idem, Phys. Essays, (2003) (in print).

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Les Principes Classiques de CED Indépendant sont rappelés à la communauté physique. Un nouveau développement: Les Principes Fondamentaux de CED Etendu sont discutés.