

## Classical Electrodynamics

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Physical reality is unique in all its manifestations. Although the number of these manifestations can be uncountable each one is unique.

Example: A physical point P on a physical plane (sheet of paper). This point can be described mathematically by two real numbers x and y. Each real number has its own mathematical identity. We have a **paradox**: one physical identity (point P) is described by two mathematical identities (x, y). But we should use only one mathematical identity to describe a single physical identity. We have to deny the mathematical identities of the real numbers x and y (transfer them to the category "mathematical accessories") and say that they together define some single math identity: two dimensional supernumber (in the category - "mathematical objects").

If we resolve this paradox (we have to develop a theory of supernumbers and use it), we will get the Special Theory of Relativity. No postulates needed! Indeed: what is the practical result of SR (disregard all sensational thought experiments)? The result is that we now have to describe all physical values by 4-dimensional form - events (4 coordinates), 4-vectors, 4-tensors,...and there is no way around it!

Once upon a time I made "a big discovery":

I found out that geometry is not mathematics, but theoretical physics, the mathematical side of which nobody studied separately!

Riemann is as close as you can get to the study of the mathematical side of Geometry, but even he did not dare to call things by their own names. He called it Geometry while actually it is a study of supernumbers. The most important property of these mathematical objects is that they carry identity. Unfortunately, this meaning was lost by contemporary mathematics which uses the term Geometry everywhere where the metric tensor is present. The term "supernumber", that I use, may be not the right one, but we have to figure out some wording for the meaning I am trying to convey. May be "number category" will be better -- future will decide.

### The Philosophy of Theoretical Physics:

Mathematics is the science of numbers. Each number has its own individuality (differs from another number). These numbers exist nowhere but in our minds. The physical reality is an infinite number of very definite things - space (event) or an electromagnetic field that is connected to this event - very definite and unrepeatable. Classical theoretical physics confirms we can describe physical reality with the help of numbers. Geometry is not mathematics but theoretical physics because the objects of the study of geometry are not numbers but physical objects - points, lines, triangles. These objects are usually at rest and we use three-dimensional supernumbers for their description. Here is the root of the difficulty in understanding special relativity: 4-dimensional supernumbers appeared to be more suitable for description of physical reality than 3-dimensional. The coordinates of the point are the mathematical components of a corresponding supernumber (something like separate digits of a real number). I think it is senseless to try to "measure" them as it was done by Einstein - all 4 coordinates ought to be prescribed with a point.

The concepts, like points (and everything that can be built up from them), scalars, vectors, tensors...- are the different categories of supernumbers. They are the mathematical objects -- carriers of math. identity (like real number in usual mathematics). While everything else -- coordinates, their axes, covariant (contravariant) components are pure mathematical concepts that serve to describe the "objects" (corresponding to - digits, points, comas,...). So, the commonly used expression "covariant vector  $A_k$ " must be read: "covariant components of the vector A". Vector A does not change with the transformation of coordinates. It changes only its description ~

components. I.e. it changes mathematics, physics remains unchanged. The coordinate transformations for the supernumber correspond to changing a real number between different systems of representations (binary, octal, decimal).

The physical objects can be described by mathematical objects. Nothing else can be used. The mathematical objects shouldn't be understood through their physical meaning. They should be available before we describe a physical reality. Previously, it was assumed that Riemann geometry is necessary only for curved space. Now the study of supernumbers (mostly Riemann geometry) becomes mandatory for every physicist. Whether space is curved or not - the meaning of a supernumber remains unchanged and requires introducing arbitrary coordinates and metrics.

Now CED is extended to inside of the elementary particle!

As it is well known, CED is based upon Maxwell's Equations of Electromagnetic Field and the equation of Relativistic Dynamics of Material Point (the last was introduced by Newton and corrected to relativistic form by Einstein).

Let us replace the equation of Dynamics of Material **point** by the equation of **field** Dynamics of Electric Currents. With that we lose completely the concepts of local mass, acceleration, and force. Still, we hope not only derive the equation of Dynamics of Material Point as a "global" object, but also take a look inside the Elementary Particle in the framework of CED.

### Lagrange Density in Classical Electrodynamics

The tensor of the EM field,  $F_{mn}$  and vector-potential  $A_m$  represent the very same physical reality. In classical physics numerical representation of physical reality is unique. The choice falls on  $F_{mn}$  because  $A_m$ , in addition, is not unique. That means, in particular, that a Lagrange density, if it has physical meaning, should not contain  $A_m$ . The classical theory of the electromagnetic field as it is given in the book Landau "Theory of Field" uses  $A_m$  in a Lagrange density. It is arranged so that it will be convenient later to build a quantum theory. It is the consequence of this arrangement, for example, that if we do not have a term current \*A then the fields are "not interacting". We then change the lagrange density and the variation method.

(Here and further we identify:  $F_{mn}$  - covariant tensor of electromagnetic field,  $F^{mn}$  - contravariant,  $j_m$  - covariant vector of current density,  $j^m$  - contravariant).

Maxwell's equations consist of two halves - the equation of currents:

$$F^{mn}{}_{|n} + 4\pi j^m = 0 \quad (1)$$

and the symmetry equation:

$$F^{*mn}{}_{|n} = 0 \quad (2)$$

where  $|$  denotes covariant derivative. When we variate action one of these equations should be a condition. Landau introduces the potential  $A(m)$  before the variation (see p.101). That makes (2) fulfilled before the variation. But we use the method indefinite Lagrange factors as a conditional variation procedure. Let us take a Lagrange density:

$$L = -(1/16\pi) F_{mn} F^{mn} - (2\pi/k^2) j_m j^m \quad (3)$$

where  $k$  is the constant of the theory. Let us build,

$L' = L + B_m F^{*mn} |_{,n}$  where  $B_m$  - 4 indefinite Lagrange factors.  
 $F^{*mn} = (1/2) e^{m nab} F_{ab}$  - dual tensor.

After varying the field, according to the Euler equation, we get

$$F^{mn} = -4\pi e^{m nab} B_{,ab}$$

From here, using the consideration of symmetry, we get:

$$F^{mn} |_{,n} = 0.$$

I.e. we are getting (1) as the result but with zero currents. The variation of currents according to Euler's equation also gives:  $J_m = 0$ . We get the vacuum.

Contrary to Landau we take (1) as a condition before variation. We build:

$$L' = L + A_m [j^m + (1/4\pi) F^{mn} |_{,n}]$$

where  $A_m$  are 4 other indefinite Lagrange factors. The variation of field gives:

$$F_{mn} = A_{,n|m} - A_{,m|n} \quad (4)$$

From here we are getting (2) as a result. The Lagrange factors  $A_m$  (which definitely look like potential) we have to choose so that the condition (1) is satisfied:

$$g^{nk} [A_{,n|m;k} - A_{,m|n;k}] + 4\pi j_m = 0 \quad (5)$$

where  $g^{nk}$  is a contravariant metric tensor (Note: D'alambert's operator  $\square A_m = -g^{nk} A_{,m|nk}$ ). These results are valid in all of space. We can, in addition, vary the currents but we have to know in what part of space we are. In a vacuum the currents are zero and we can not vary them. Inside the elementary particle we can vary the currents keeping the variation zero at the boundary of the particle. The Euler equation gives:

$$4\pi j_m = k^2 A_m \quad (6)$$

This along with (5) leads to:

$$\square A_m + k^2 A_m = 0 \quad (7)$$

which holds only inside the particle (note that we have the sign + in this equation while the Klein-Gordon equation has a minus sign). In a vacuum  $j(m)$  is zero and equations (4), (5), and (1) are holding. We have:

$$\square A_m = 0 \quad (8)$$

The Lagrange factors  $A_m$  inside particle and in vacuum correspond to different space and different variational procedures and so, it is not necessary for them to be continuous on the boundary of the particle. It is useful to remember that EM potential is nothing less than 4 Lagrange factors of some conditional varying procedure - no physical meaning, no uniqueness is necessary.

### Energy-Momentum Tensor

From the Lagrange density (3) we can get the energy-momentum tensor using the variation procedure formally close to that described in the Landau book on p.345 but with a different internal meaning. Instead of an arbitrary small transformation of coordinates, we vary only the metric tensor:

$$dg^{mn} = g^{na} \xi^m_{|a} + g^{mb} \xi^n_{|b}$$

where  $\xi^m$  is a small arbitrary vector. This variation of metric preserves the curvature of the space. So, if original metric corresponds to a flat space, then varied metric will represent also a flat space. Please note that we vary contravariant metric and keep our physical fields covariant in this variational procedure. As a result we are getting energy-momentum tensor:

$$T^{mn} = -(1/4\pi) g_{ab} F^{am} F^{bn} + (1/16\pi) F_{ab} F^{ab} g^{mn} + (2\pi / (c^2 k^2)) [-2j^m j^n + j^a_j g^{mn}] \quad (9)$$

and the conservation equation: (Dynamics):

$$[(ck^2)/(4\pi) F_{mn} + j_{m|n} - j_{n|m}] j^n = 0 \quad (10)$$

This is a vector equation (index m is free). The equation is a nonlinear equation (2d order with respect to unknown functions, 10 of them are independent). The whole PDE system consists of (1), (2), and (10). As it is known from mathematics, the characteristics of the system of partial differential equations are the major players in obtaining the solutions of the system. Here, in addition to the wave fronts that move with the speed of light (Maxwell's equations), we have 3-dimensional surfaces to which vector  $j_m$  is tangent. These are the surfaces of the elementary particles!

Attached to this paper is the file: pdemo.exe The execution of this file gives a graphical images of the classical models of Electron, Proton, and Neutron together with some numerical data. These models are not very good but are still the solutions of the above mentioned equations.

[Download pdemo.exe](#)

## CED

The first order PDE system (1), (2), (10) is nonlinear (second order nonlinearity) system for 10 unknown independent functions. This form of the system is a canonical form because it can be considered as a source point of the theory under consideration.

How do we get solutions? The Dynamics equation (10) hints that either square bracket equals zero and currents are different from zero (inside EP), or currents are zero (vacuum). This two regions are separated by characteristic of the secoto (7). In the vacuum the potential satisfies (8).

## Ideal Particle

Now, when the system, (the solutions of which we are going to look for), is written, we want to think that mysticism is completely excluded. It is obvious that the system describes everything: particles, their movements, their interaction - a certain kind of "alive" world, but, maybe a world different from the real world. We do not know that until we get all the solutions we are interested in. It is also obvious, that we simply are not able to get 99.9% of these solutions due to the mathematical difficulties. I, naturally, hope that the "world of solutions" corresponds to the real world. I also have to say, that having much experience in attempts to get the solutions, I came to the conclusion that mysticism connected to the characteristics of the system was not removed in full. The characteristic is the disruption of a solution that does not contradict the system. But the surface of disruption is often impossible to choose so that the system is fulfilled everywhere on that surface. What laws are in control here we still do not know. It is clear from the above that 0.01% of the solutions are possible. One especially

stands out: The Ideal Particle. In spherical coordinates:

$$j_0 = a \sin(z)/z, \quad 0 < z < z_1; \quad j_0 = 0, \quad z > z_1; \quad z_1 = \pi, 2\pi, 3\pi, \dots \quad (11)$$

where:  $z = kr$ ,  $a$  - constant,  $j_m = 0$ ,  $m = 1, 2, 3$ . I exclude the electric field - it is clear that it can be found, as well as the integration can be done and the mass and full charge can be found. This solution is an exact static equilibrium - there are no violations of the system. It appears, though, that this equilibrium is unstable. It is rather easy (after a Lorentz transformation) to get an exact solution for the inertially moving IP. Again, we can integrate and get the full energy and linear momentum - they satisfy to the well known relativistic relation:

$\text{mass}^2 = \text{energy}^2 - \text{momentum}^2$ . The next problem will be to include spin in the solution. The classical models of electron, proton, and neutron, which are given in the Demonstration, possess the mass, charge, spin, and magnetic moment that correspond to the table data, but they are obtained with the violations of the system on the surfaces of disruption.

### More about IP

The Ideal Particle represents by itself something absolutely exceptional: it satisfies the system in all space which is homogeneous and isotropic (I mean, that the surface of disruption - the surface of the particle is defined only by the initial data but not by any inhomogeneity of the space). The electrical field goes to zero at infinity. IP possesses electric charge and mass. On the surface of disruption the derivative from the charge density has a jump but the electric field is continuous. I can tell, that the IP is what physicists were looking for since the beginning of the last century (the corresponding statements by Einstein do exist).

The IP has a definite historical contact: the "problem of 4/3". The book of Landau omits this problem. The problem is described by Feynman in "Lectures on Physics" vol.2, chapter 28-3 "Electromagnetic Mass". It appears that the electromagnetic mass of the vacuum field of the electron (I would call it an inertial mass) equals to 4/3 of the rest mass of the same field. This fact caused a "big confusion" at the beginning of the last century. The IP also has a vacuum field and the inside region where we have a charge density and electric field. The rest mass in this region is zero, but the inertial mass is -1/3. As a result of that, the full inertial mass is equal to the full rest mass (Note that a negative energy density is normal in this theory).

### Spin

I've managed to prove a very interesting theorem: If we have a conserving energy-momentum tensor and a field that "good" decrease at infinity then the global spin of this field can be different from zero only in the case if this field actually depends on time. If the field is static then the spin is equal to zero. That means that the particles with spin must at least oscillate.

Indeed: we found two static solutions of the above mentioned system:

1. The particle in a constant electric field that possesses an electrical dipole moment (Dipole Particle DP). the dipole moment is proportional to the electric field.
2. The particle in a constant magnetic field that possesses spin (Static Spin Particle SSP). The spin is proportional to the magnetic field. Both particles do not satisfy to the condition of the theorem at infinity. This allows SSP to have a legal static spin.

There exists a different way: The static particle that is "good" at infinity but violate the system on the boundary of disruption. To obtain a spin in this way we can introduce a density of circular current directed along  $z$ -axis in spherical coordinates:

$$j_3 = b[\sin(z)/z^2 - \cos(z)/z] \sin(\theta) \quad (12)$$

where:  $z = kr$ ,  $b$  - a constant. The magnetic field can be found inside and in the vacuum, but in order to be continuous on the boundary of disruption (and decrease at infinity), the boundary has to be at  $z = \pi, 2\pi, 3\pi, \dots$

The charge density is zero at these points, but the density of circular current is different from zero. This jump of the current violates the system.

If we can (assumably) violate the system by cutting a density of circular current, then we can also cut the charge density not only at points  $\pi$ ,  $2\pi$ ,  $3\pi$ , ... but where we like. In this case we will get the static classical models for all elementary particles (see the Demonstration).

The problem of Spin also has a historical contact: Somewhere at the beginning of the last century (I forgot where) physicists calculated the energy of the vacuum magnetic field of the electron. They found that this energy is 400000 times greater than the rest mass of the electron. So it was also a confusion!

In our classical model of the electron the magnetic energy of the inside region is exactly equal to the vacuum energy and has a minus sign so that the mass of the electron is defined only by its electric charge and field.

### **Cylindrical Particles - Photons**

If we cut the electrical charge density and density of circular current violating the system on the boundaries of disruption, then we can get solutions for the particles that move with the speed of light and have a form of a cylinder with "bottom" and "cover" (the axis of the cylinder coincides with the direction of propagation). This particle is accompanied by the EM wave in vacuum. The frequency of the wave correspond to the length of the cylinder. These particles are nothing less than classical photons. Here we have a jump forward compared to QT: we have two sorts of photons in CED while QT gives only one kind of a photon.

1. Halfparticle-halfwave (laser's beam photon): This photon has a full rotating symmetry around the axis of the cylinder. It is accompanied by the wave in vacuum that decreases its intensity at infinity but not fast enough. The spin density is different from zero only inside the cylinder. The energy inside the cylinder equals the half of the product of spin on frequency. The energy of the vacuum wave is infinite (has a logarithmic divergence). This wave has full rotational symmetry and can not exist without the particle. The electric field in the wave and inside the particle and has round polarization.

2. Nuisance Particle. This particle can be "combined" with the usual plain wave which does not decrease at infinity at all. But this combination is free - the plain wave can exist without the particle. The spin density is different from zero inside the cylinder and the energy of the particle in the absence of plain wave equals to the product of spin on frequency (the particle has its own fast decreasing field in vacuum). The polarization is also circular. This is the usual photon which can be emitted and absorbed by the atoms.

Both classical photons have the spin that is directed along or against the momentum but they have a different picture of directions of electric and magnetic fields. The nuisance particle has its field vectors directed at the same angle with the X-Y axes (as in plane wave), while the intensity depends on the radius of cylindrical coordinates. This is why this particle is easily "combined" with a plane wave (with this combination the parameters of the particle slightly change depending on the intensity of the plane wave. For example: the energy inside the cylinder goes down when the intensity of the plane wave goes up - the particle more likely will be found in the more intense plane wave if it has a choice). The vectors of the electromagnetic field of the Halfparticle have rotational symmetry. The Halfparticle was named so because it has finite spin as any particle and it has infinite energy as any plane wave. But there is a big difference: the intensity of the plane wave is equally distributed over the plane while the intensity of the vacuum wave that accompanies the halfparticle is concentrated around the cylinder (the field is inversely proportional to the radius).

I'm positive that the Halfparticle is unknown to quantum theory.

### **Instability of the Particles and some Cosmic Problems**

It was mentioned, that IP is an unstable equilibrium. One can think that the whole theory will fall apart if we take this into account.

Note 1: The concept of instability is not applicable to Photons.

Note 2: There were attempts to find out what happens if IP gets some small perturbation. If the perturbation has a spherical symmetry (which is almost impossible in reality) then there was derived an integral equation for the motion of the spherical boundary. The analytical solution for the small movements of the sphere gives us oscillations with proper frequencies. If the movements are not small - we can get a numerical solution which shows that the oscillations grow fast in amplitude. But this is true only if the amplitude of velocity remains below of the speed of light. What happens when the speed of the sphere approaches the speed of light we were not able to get directly but it is clear from general consideration that radiation in full spherical symmetry is impossible.

Can the charge density spread homogeneously over the whole world? - No: the energy-momentum tensor shows that the energy of distributed charges less than the energy of concentrated charges. Since there is no place for energy to go - we are coming to the conclusion that the sphere will be oscillating stationary with the maximum speed just below of the speed of light. But the particle won't fall apart.

Now a more real situation: the perturbation is arbitrary. Here we use only general considerations. Since there is no spherical symmetry, radiation becomes possible. The radiation will take energy and the particle will be expanding. Indeed, if we take the series of ideal particles with the same charge but with different radii  $r_1 = \pi, 2\pi, 3\pi, \dots$ , then their masses will be:  $m, m/2, m/3, \dots$ . We have to conclude that all the energy will be radiated in a form of EM waves and the charge density will spread uniformly over the whole universe!

Now attention: This will happen if we have only one particle in the whole universe. If we have many particles then they will create some "background" radiation and it creates a possibility for the particle to absorb. In this case it will come to a steady state where radiation is equal to absorption and the particles will be "saved". It is obvious, that the frequency of this radiation will correspond to the size of the particles. It is also obvious, that we can not "see" this radiation because this is the vacuum electromagnetic waves but not photons. Unfortunately, we learn only how to register waves only for radio frequencies. Above that we register only photons. The vacuum waves slip through our fingers!

And one more surprise: if high frequency background radiation is so strong, then we have a chance to explain gravitation on the account of incoming background radiation. Let us take a galaxy. The intensity of background radiation should decrease towards the center of the galaxy and so should decrease the "effective gravitational constant". Some simple assumptions allow us to show that:

1. The problem of "lost mass" can be easily solved without the introduction of hypothetical interstellar matter.
2. If in cosmic space the intensity of background radiation is equal in all directions then the galaxy will be spherical. The flat galaxy (like ours) exists in the space where incoming background radiation is equal in all directions in the X-Y plane but it is less in the perpendicular direction (it is clear that the orbital movement in the plane where background radiation is not equal in all directions will result in loss of energy). This assumption allows us to explain why a flat galaxy turns to spherical shape closer to its center.
3. If incoming background radiation equal in all directions in the X-Y plane but is bigger in a perpendicular direction then the galaxy will be also flat but without spherical formation in the center (this kind of galaxies also exist).
4. If we take a very sensitive gravimeter, then, during the Earth's rotation, we can get a record that can not be fully explained by the movements of Moon, Sun, and all the Planets. We have to introduce hypothetical masses beyond the Solar system. The anisotropy of background radiation will explain this very easily.

I have proof for many of the above statements, so you are welcome to ask questions and present contradictory opinions.