the general science journal

The Source, the Field or the Metric?

Petr Křen © 2006
pkren@cmi.cz

Physical interpretations were transferred from a source picture to a field and metric picture in the 19th and 20th century respectively. But we must return to the earlier method for better comprehension.

It is very well known that the stationary path is given by,

$$\delta\int ds = 0$$

and in the general relativity metric interpretation by,

$$\delta\int \sqrt{g_{ij}d\dot{x}^i d\dot{x}^j}\,du = 0 \qquad \text{or} \qquad \delta\int g_{ij}d\dot{x}^i d\dot{x}^j du = 0$$

A solution of the last equation is found by using the Euler-Lagrange equations (as in the “classical” mechanics source picture).

$$\frac{d}{du}\left(\frac{\partial L}{\partial \dot{x}^i}\right)-\left(\frac{\partial L}{\partial x^i}\right)=0 \qquad \text{with } L \equiv g_{ij}d\dot{x}^i d\dot{x}^j$$

This solution is the general relativity geodesic equation that can be written in the form:

$$g_{hj}\ddot{x}^i + \left(g_{hi,j} - 1/2\cdot g_{ij,h}\right)\dot{x}^i\dot{x}^j = 0$$

For a particle moving slowly in a weak stationary gravitational field, we can derive a metric that corresponds in the classical limit to Newtonian solutions. The simplest gravitational solution in polar coordinates is the Schwarzschild metric:

$$ds^2 = \left(1-\frac{r_S}{r}\right)c^2dt^2 - \left(1-\frac{r_S}{r}\right)^{-1}dr^2 - r^2\left(d\theta^2 + \sin^2\theta\cdot d\phi^2\right) \qquad \text{where } r_S \equiv \frac{2GM}{c^2}$$

a corresponding solution to the Euler-Lagrange equations is the relativistic Binet equation:

$$\frac{d^2u}{d\phi^2} + u = \frac{r_S c^2}{2h^2} + \frac{3}{2}r_S u^2 \qquad \text{where } u \equiv \frac{1}{r}$$

and h is the angular momentum per mass unit.

Using this metric, we can predict all effects that were measured to support general relativity, such as the red shift, the Shapiro time delay, the deflection of light and the perihelion shift. The horizon in this metric is equal to r_S. But we can add higher order terms into this weak field metric to obtain solutions undistinguishable using current instruments. For example:

$$\left(1-\frac{r_S}{r}\right) \rightarrow \left(1-\frac{r_S}{r}+\left(\frac{r_S}{2r}\right)^2\right) = \left(1-\frac{r_S}{2r}\right)^2$$

and the Binet equation becomes,

$$\frac{d^2u}{d\phi^2}+u = \frac{r_S c^2}{2h^2}+\frac{3}{2}r_S u^2 - \left(\frac{r_S}{2}\right)^2\left(\frac{c^2}{h^2}u+2u^3\right)$$

(Terms with h are equal to 0 in the case of light.)
However, the horizon is now $r_S/2$. Note that it is possible to select metrics where there is no horizon. So we cannot determine more within the scope of recent observations.

Compare with Reissner-Nordstrom metric solution:

$$\frac{d^2u}{d\phi^2}+u = \frac{r_S c^2}{2h^2}+\frac{3}{2}r_S u^2 - \left(\frac{GQ^2}{4\pi\varepsilon_0 c^4}\right)\left(\frac{c^2}{h^2}u+2u^3\right)$$

But we can introduce another metric with

$$g_{44} = \frac{\left(\frac{T+V}{m_0 c^2}\right)^2}{1+\left(\frac{\hbar}{J_\phi}\right)^2}$$ where J is the angular momentum

and $$-\frac{T}{m_0 c^2} = \left[1+\left(\frac{\frac{V}{c}r}{J_r+\sqrt{J_\phi^2-\left(\frac{V}{c}r\right)^2}}\right)^2\right]^{-\frac{1}{2}} = 1-\frac{1}{2}\left(\frac{Vr}{cJ}\right)^2+\cdots$$ is close to 1

because $J \doteq n\hbar$ where $n \approx \frac{r}{r_{particle}} > \frac{r_S}{r_{particle}}$.

For $$\frac{V}{m_0 c^2} = -\frac{GM}{c^2}\frac{1}{r}$$ we obtain $$g_{44} = 1-\frac{r_S}{r}+\cdots$$
and solution has the same Schwarzschild Binet equation used for general relativity tests.

(Angular momentum is always stronger (no horizon, no singularity) thus a collapsing object must rotate producing a disk and jets. This momentum grows weak towards a "fundamental state" as entropy is growing, but an object does not collapse. Also a charged particle does not explode and it results from an unknown, non-linear behaviour.)

For $V = \hbar c Z\alpha \frac{1}{r}$, where Z represents the number of elementary charges (Z=-1 for the hydrogen atom) and r, the distance from the centre of gravity (instead of a nucleus position in this approximate version, neglecting particle structure), we can derive the Binet equation for a hydrogen-like atom in this metric convention as per Sommerfeld (in the source picture)

$$\frac{d^2u}{d\phi^2} + u = -\frac{Z\alpha}{n_\phi^2}\frac{E_n}{\hbar c} + \frac{(Z\alpha)^2}{n_\phi^2}u$$ with quantum numbers $n = n_r + n_\phi$.

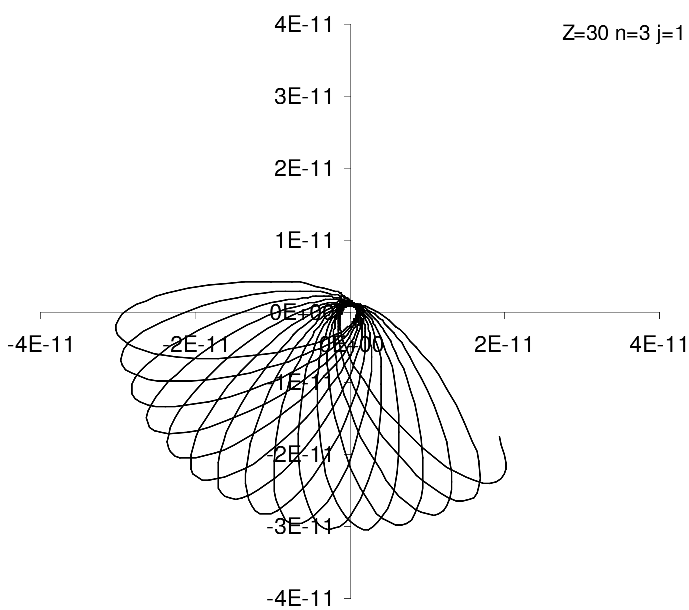
where $$\frac{E_n}{m_0 c^2} = \left[1 + \left(\frac{Z\alpha}{n_r + \sqrt{n_\phi^2 - (Z\alpha)^2}}\right)^2\right]^{-\frac{1}{2}}$$

All whole numbers in any picture are derived from cyclic coordinate conditions (Bohr, interference) as harmonics or a non-linear field presumption (but only the fundamental or linear is stable in the case of a single particle/body).

This Sommerfeld source solution of electron motion corresponds to the same energy states as the Dirac distribution/field solution (transformation between these solutions is provided by the wave equation and the electric field corresponds to wave-function distribution [1]). The orbit with an axis ratio of approximately $\frac{n_\phi}{n} = \frac{n_\phi}{n_\phi + n_r}$ has a relative "periprotonion" shift per orbit given by the expression

$$\frac{\Delta\phi}{2\pi} = \frac{1}{\sqrt{1 - \frac{(Z\alpha)^2}{n_\phi^2}}} - 1 \doteq \frac{(Z\alpha)^2}{2n_\phi^2}.$$

Compare this with the Thomas precession equation: $\frac{\omega_p}{\omega} = \frac{\Delta\psi}{2\pi} = \frac{1}{\sqrt{1-\beta^2}} - 1$



The closest approach to the centre of gravity is given by

$$r_{\min} \doteq \frac{\hbar}{m_0 c}\frac{n_\phi^2-(Z\alpha)^2}{Z\alpha}\left[1+\sqrt{1-\frac{n_\phi^2-(Z\alpha)^2}{\left(n_r+\sqrt{n_\phi^2-(Z\alpha)^2}\right)^2}}\right]^{-1}$$

and this approach has extreme values for $n_\phi = 1$ (s-state), thus the nucleus position correction (the Lamb shift) is large in this case.
If we incorporate effects related to nucleus structure ("producing" field) we obtain hyper-fine splitting.

And more: Solving an equation describing Fermat's principle in form [2],

$$\delta\int n\sqrt{2T}\,dt = 0 \qquad \text{or} \qquad \delta\int \varepsilon_{ij}\,d\dot{x}^i\,d\dot{x}^j\,dt = 0$$

we are able to interpret all effects supporting general relativity mentioned above in terms of the optical properties of the medium. And [3] we can define a special energy-momentum tensor that enables calculations equivalent to Einstein's equations (also weak field limits) with a non-geometrical approach, thus a transformation between the field picture and the metric picture is only formal.

[1] V.M. Simulik and I.Yu. Krivsky: Slightly generalized Maxwell classical electrodynamics can be applied to inneratomic phenomena, Annales de la Fondation Louis de Broglie, Volume 27 no 2, 2002, p. 303, http://www.ensmp.fr/aflb/AFLB-272/aflb272p303.pdf
[2] B. H. Lavenda: Three Tests of General Relativity via Fermat's Principle and the Phase of Bessel Functions, arXiv:math-ph/0310054 v1, 25 Oct 2003
[3] S.V. Babak, L.P. Grishchuk: The energy-momentum tensor for the gravitational field, arXiv:gr-qc/9907027 v2, 30 Oct 1999

Errata:
Petr Křen, Notes on Relativity http://wbabin.net/science/kren.pdf, Received Aug. 19, 2005, page 2
"... it is *not* necessary to give a name ..."