

One way and two way speed of light

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The math mistakes made by SR (Special Relativity) texts while dealing with two way and one way speed of light will be dealt with and the link between Newtonian and Einsteinian physics shown.

In considering light travelling to a mirror and bouncing back, many SR texts write the equation:

$$2t = L/(c-v) + L/(c+v) = 2Lc/(c^2 - v^2) = 2L/(c(1 - v^2/c^2)) \quad (1)$$

We have to consider what this equation means:

First of all I want to split this equation (1) down and just talk of the first part:

$$2t = L/(c-v) + L/(c+v) \quad (2)$$

Light is travelling a distance L between a source and a mirror

Source *-> | Mirror
.....<-----distance L----->

The total distance travelled is 2L

If we have a clock at the source then time for first part of the journey is t
for the second part of the journey (return trip) is t
total time for light going to mirror and back is 2t

Einstein considered the problem of synchronizing clocks; if we had a clock at the mirror how would we know it was synchronized with the clock at the source- He thought we could not ensure perfect synchronization so it was best to have just one clock at the source then it would measure time for the two way journey.

i.e. we have the two way speed of light from this setup from using just one clock and measuring the distance 2L and time interval 2t.

If we could ensure perfect synchronization of a clock at the mirror with the clock at the source then we could measure distance L and time interval t and get the one-way speed of light.

So this is what equation (2) means by $2t$ and $2L$; total time of light for round trip = $2t$ and total distance = $2L$.

Looking again at equation (2):

$$2t = L/(c-v) + L/(c+v) \quad (2)$$

we have extra terms $(c-v)$ and $(c+v)$

I shall get to those terms in a moment.

If light travels at speed c in forward direction and speed c in back direction then we have:

$$L = ct \text{ in both directions} \quad (3)$$

(light moves $+c$ in forward and $-c$ in backward direction, but distance is same in both directions)

total distance travelled by light = $2L$

total time = $2t$

speed of light = c

$$2t = 2L/c \quad (4)$$

if we subst $v = 0$ into equation (2) then that is what we have

$$2t = L/(c-0) + L/(c+0) = 2L/c \quad (5)$$

i.e. equation (4) equals equation (2) for $v = 0$

Equation (4) is dealing with the rest frame observer – i.e. the observer that is stationary with respect to the source and the mirror.

If the source and the mirror are moving with velocity v with respect to an observer then he will see the $(c-v)$ and $(c+v)$ terms, where first the light goes with the velocity v of the moving source and mirror [giving $(c+v)$] and then on return trip goes against the velocity v of the source and mirror [giving $(c-v)$].

The observer watching the moving source and mirror travelling at velocity v sees first the light travel at $(c+v)$ and on return sees it travel at $(c-v)$.

Now because of the muddle created by people saying the speed of light is constant, they say no-no that cannot be right; the speed must be c in both direction not $(c+v)$ and $(c-v)$. But that is unfortunately them confused by the subject.

If we think in terms of distances then light travels distance $(c+v)t$ in the forward direction; if light travels ct in time t then the mirror has moved vt in that time t , so that light has to travel distance $(ct+vt)$ to get to the moving mirror. And on the return trip the source moves towards the light and the distance the light has to travel is ct less vt , namely $(ct - vt)$.

Both outward journey and return journey takes same time interval t according to observer watching the moving source and mirror; that means the light has to travel faster $(c+v)$ and slower $(c-v)$ on the return trip.

i.e the one way speed of light is different for the forward journey to the return journey, and that is according to the observer aware of the speed v . A person moving with the source and mirror is not aware of speed v (or rather has $v = 0$) and has one way light speed the same in both directions.

For the observer aware of non-zero v hence having $(c+v)$ and $(c-v)$, the mean speed of light is:

$$\text{sqrt} [(c+v)(c-v)] = \text{sqrt} (c^2 - v^2) = c \text{sqrt} (1 - v^2 / c^2) \text{ (positive solution) (6)}$$

i.e the two way speed of light for observer aware of non-zero v has it as (6)

Max Born was aware of this, but after being aware of that did his maths wrong. [1]

Look again at (2):

$$2t = L/(c-v) + L/(c+v) \text{ (2)}$$

the use of L is incorrect, I will replace by L_1 and L_2

$$2t = L_1/(c-v) + L_2/(c+v) \text{ (7)}$$

$$L_1 = (c-v)t \text{ the return trip (8)}$$

$$L_2 = (c+v)t \text{ the forward trip (9)}$$

subst (8) and (9) into $L_1/(c-v) + L_2/(c+v)$ gives:

$$(c-v)t/(c-v) + (c+v)t/(c+v) = 2t \text{ as required (10)}$$

The equation (2) is erroneous. What happened was the results of the two observers was confused into one equation.

The observer stationary with respect to the source and mirror has $L = ct$ in both forward and return trips so that $2t = 2L/c$.

But the observer seeing non-zero v has (8) and (9).

The results of what both observers see has got confused into one equation creating an invalid equation (2).

Proceeding now with the correct equation (7) we have:

$$2t = L_1/(c-v) + L_2/(c+v) \quad (7)$$

$$L_1 = (c-v)t \text{ the return trip} \quad (8)$$

$$L_2 = (c+v)t \text{ the forward trip} \quad (9)$$

for the observer aware of non-zero v .

This observer has the two way (mean) speed of light as:

$$\text{sqrt} [(c+v)(c-v)] = \text{sqrt} (c^2 - v^2) = c \text{ sqrt} (1 - v^2 / c^2) \text{ (positive solution)} \quad (6)$$

so his claim is that in time interval $2t$, the two way (mean) speed of light travels distance $2L$ as:

$$2L = 2ct \text{ sqrt} (1 - v^2 / c^2) \quad (11)$$

we can drop the "2":

$$L = ct \text{ sqrt} (1 - v^2 / c^2) \quad (12)$$

Now this equation is consistent with Newtonian physics. The mean speed of light (6) was derived still based on Newtonian physics.

The speed of light is variable in Newtonian physics, so let us write

$$c' = c \text{ sqrt} (1 - v^2 / c^2) \quad (13) \text{ then equation (12) is:}$$

$$L = c' t \quad (14)$$

However due to the confused state of mind of Einstein he interpreted time as variable so let us write

$$t' = t \text{ sqrt} (1 - v^2 / c^2) \quad (15) \text{ then he writes (12) as:}$$

$$L = ct' \quad (16)$$

Both (14) and (16) are interpretations of equation (12), so both Newtonian physics and Einstein's SR have the same maths at their root.

Of course other confusions were made in SR and GR (General Relativity). But when the maths mistakes are corrected then mathematically the roots of Newton and Einstein are the same theory.

The maths mistakes with relativity texts seem to have their roots with Einstein and his associates like Max Born, and instead of correcting those mistakes, relativity texts just keep repeating them verbatim. The issue with relativity often is about what two different observers observe and the maths from these two different frames gets messed together incorrectly. One mistake leads to another mistake until it is a large collection of mistakes.

I shall now go over the same ground again in summary:

Summary

$$c' = c(1 \pm v/c)$$

i.e. plus or minus v/c

which is claimed to be back and forth

the forward part is:

$$c'_1 = c(1 - v/c)$$

and the backward part is

$$c'_2 = c(1 + v/c)$$

we multiply c'_1 and c'_2 and get

$$c'_1 * c'_2$$

we square root this and we have mean speed of light

$$\sqrt{c(1-v/c)c(1+v/c)} = \sqrt{c^2(1 - v^2/c^2)} = c \sqrt{1 - v^2/c^2}$$

(positive solution)

many SR texts have the equation:

$$2t = L/(c-v) + L/(c+v) = 2Lc/(c^2 - v^2) = 2L/(c(1 - v^2/c^2))$$

but that has two different quantities being represented by L, it should have used L_1 and L_2

forward journey is $(c-v)t$

backward journey is $(c+v)t$
this is from an observer observing the v

so total distance traversed = $(c-v)t + (c+v)t = 2ct$

so the equation should have been

$$2t = (c-v)t/(c-v) + (c+v)t/(c+v) = 2t$$

From earlier using mean speed of light $c' = c \sqrt{1 - v^2 / c^2}$
(positive solution)

we have distance = $c't = c \sqrt{1 - v^2 / c^2} t$

But Einstein would not accept that speed of light varied so instead of c'
had time vary i.e had t'

so this equation got written as

$$ct' \text{ where } t' = \sqrt{1 - v^2 / c^2} t$$

in GR Einstein went back to variable lightspeed just to confuse things
further.

Really if Einstein went by Newtonian Physics it would be variable lightspeed
plus universal time; but he decided to think lightspeed was constant and
made time variable.

the difference being $c't$ by Newton

ct' by Einstein

by Newton $c't = c \sqrt{1 - v^2 / c^2} t$ (a)

by Einstein $ct' = c \sqrt{1 - v^2 / c^2} t$ (b)

equations (a) and (b) are equal.

Common to both Einstein and Newton after mistakes have been corrected is the
equation:

$$c \sqrt{1 - v^2 / c^2} t$$

By Newton we would interpret this as variable light speed and universal time

By Einstein SR we would interpret this as constant lightspeed and variable
time.

But the maths is the same in both theories.

There are of course other maths mistakes.

But once the maths mistakes are corrected, both theories are mathematically the same thing.

The difficulty with Einstein +co is that they made too many mistakes and it raises the issue of where to pickup from their mistakes. In the case of one-way and two-way lightspeed, they made mistakes that might be considered fundamental before even dealing with that issue.

References and notes

[1] Einstein's Theory of Relativity, Max Born, Dover USA 1962 (revised and enlarged version of work published in 1924) – Max Born devotes most of his book building upon the mistake highlighted in this article. What he correctly identifies is that dealing with light travelling the same path twice from source to mirror and being reflected back what is being dealt with is the mean velocity. He says: "It (light) traverses the same path twice, and hence it is only the mean velocity during the motion to and fro that is measured." (p 96) He also notes correctly that if we suppose that the velocity of light is not the same in both directions, then this influence will be wholly or partially cancelled in the motion to and fro. It is just unfortunate he then makes the mistake with L.

And those who came after him just followed his example.

[2] Note on mean speed of light: From my notation we have two speeds c'_1 and c'_2 in the forward direction and in the back direction.

Multiply them together we have the speed squared, square root that $\sqrt{c'_1*c'_2}$ and we have the mean speed of light; that is the mean speed of the light going in one direction and coming back. The speed forward is different to the speed back, so we take the mean of them.

In dealing with this scenario above I have glossed over a difficulty. I have been dealing with "mean speed of light"; what I call that is really the geometric mean. There is also an arithmetic mean often called average. So the issue becomes on that why deal with "mean" (geometric mean) speed of light instead of "average" (arithmetic mean) speed of light? And the answer seems to be – I don't know. It seems that Einstein and those who came after him (Einsteinians) just did it that way.

If we go for the average speed of light we have the following:

for the observation of light moving forward and back in the stationary frame:

forward distance = ct

backward distance = ct

total distance = $2ct$ and average speed of light = c

for light in moving source-mirror setup

forward distance = $(c+v)t$
backward distance $(c-v)t$

[taking $t = t'$]

total distance = $(c+v)t + (c-v)t = 2ct$

average speed of light = c

i.e. average speed of light is the same in both

For some reason was done by using geometric mean:

$$ct' ct' = c^2 t'^2$$

and

$$(c-v)t (c+v)t = (c^2 - v^2) t^2$$

Lorentz transformations then connect:

$$x^2 - c^2 t^2 = 0$$
$$x'^2 - c^2 t'^2 = 0$$

$$\text{and } c^2 t'^2 = (c^2 - v^2) t^2$$

Newtonian would have really done it instead as :

$$x^2 - c^2 t^2 = 0$$
$$x'^2 - c'^2 t'^2 = 0$$

$$\text{and } c'^2 t'^2 = (c^2 - v^2) t^2$$

But one thing to note is that whether it is

$$x^2 - c^2 t^2 = 0$$
$$x'^2 - c^2 t'^2 = 0$$

or

$$x^2 - c^2 t^2 = 0$$
$$x'^2 - c'^2 t'^2 = 0$$

its connecting two different distances together by a transformation. The distance x is different to x' . That's not really what one is supposed to do if one is being sensible, and one would not be doing that if going by average speed. But they (Einstein +co) went by mean speed; and that might have been a great big fundamental mistake from first step.