

# **Newtonian time dependent equation: synthesis of relativity and quantum physics**

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Abstract: An equation derivable from Newtonian Physics has been overlooked for hundred of years, namely the “Newtonian time dependent equation of motion”. This equation unifies equations of relativity and quantum physics from a Newtonian perspective.

This article is based upon the work of Joe Nahhas [1, 2, 3, and 4] and seeking to try to explain things in a more heuristic way.

A fundamental idea in celestial mechanics is areal velocity, it is the area swept out by the radius vector per unit time. [5]

Areal velocity is:

$$dA/dt = (r v)/2$$

Which Joe Nahhas changes by using

$$v = r d\theta/dt = r \theta'$$

$$dr = r d\theta$$

to:

$$dA/dt = (r^2 \theta')/2$$

$\theta$  = angle

$\theta'$  =  $d \theta/dt$

$r$  = radius

$v$  = speed

$A$  = area

$t$  = time

By Kepler's 2<sup>nd</sup> law – a line from the sun to a given planet sweeps out equal areas in equal time. [6]

$$dA/dt = (r^2 \theta')/2 = \text{constant}$$

Joe re-arranges to :

$$r^2 \theta' = 2 * \text{constant and calls } 2 * \text{constant as } h,$$

Areal velocity is constant:  $r^2 \theta' = h$  (Kepler's Law)

Joe explains it:

This is Kepler's law where area is the distance and  $\theta'$  (derivative of  $\theta$  = change of  $\theta$  per time).

Kepler's law says the areal velocity as swept by planets around the Sun is constant. [7]

So we have:

$$h = \text{constant} = r^2 \theta'$$

Thus

$$h = r^2(1) \theta'(1) = r^2(2) \theta'(2) = r^2(3) \theta'(3) = \dots \text{etc}$$

In general  $r$  is a function of angle  $\theta$  and time  $t$ , so we should really write:

$$r = r(\theta, t)$$

(similarly  $\theta'$  is a function of  $\theta$  and  $t$  so that should really be written  $\theta'(\theta, t)$ )

Now considering the case of time  $t = 0$ , this is the static case

for what Joe calls the Newton's classical solution of a traveler (traveling object).

i.e.

$$r = r(\theta, 0) \text{ case}$$

We can differentiate with respect to time the equation  $r^2 \theta' = h$

(note: we can have for this the  $r(\theta, t)$  case with  $\theta'(\theta, t)$  or the  $r(\theta, 0)$  case with  $\theta'(\theta, 0)$ )

Differentiation gives

$$d(r^2 \theta')/dt = dh/dt = 0$$

(h is a constant its differentiation is zero)

The differentiation of  $r^2 \theta'$  is

$$2r r' \theta' + r^2 \theta'' = 0$$

dividing by  $r^2 \theta'$

$$\text{you get } 2(r'/r) + (\theta''/\theta') = 0$$

rearrange gives:

$$2(r'/r) = -(\theta''/\theta')$$

This is a second order differential equation

The method we are going to use is separation of variable.

Now we are first going to explain that

$$2(r'/r) = -(\theta''/\theta')$$

equals  $2 [\lambda(r) + i \omega(r)]$

(Nahhas refers to this as equal to “constant or constant number” but I think he is better saying variable; I will now just refer to it as “number”.)\*\*

We proceed thus:

$$2(r'/r) = -(\theta''/\theta')$$
 is the original equation

The solution to this equation is as follows:

To make each side equal to a number.

you choose a number

you can choose a real number or a complex number.

The general solution is a complex number (i.e. 2 dimensional case)

$$a + ib$$

Choose  $a = 2\lambda$ ; with  $\lambda$  as equals  $a/2$ ; where letter  $a$  could be any number

and  $b = 2\omega$ ; with  $\omega$  as equals  $b/2$ ;  $b$  could be any number

with  $2(r'/r) = 2[\lambda + i\omega]$

So

$2(r'/r) = -(\theta''/\theta') = 2[\lambda(r) + i\omega(r)] = \text{complex number}$

then  $r'/r = \lambda + i\omega$

Now  $r' = dr/dt$

so  $r'/r = \lambda + i\omega$

becomes

$dr/dt = \lambda + i\omega = \underline{\text{complex number}}$

rearrange

$dr/r = (\lambda + i\omega)dt$

Integrate both sides

$\int (1/r) dr = \int (\lambda + i\omega) dt$

becomes:

$\ln r = (\lambda + i\omega)t + k$

where  $k = \text{some constant}$

we can choose  $k = \ln r(0)$

because for  $r(t)$   $r$  as function of  $t$ , for  $t=0$ ,  $\ln r = (\lambda + i\omega)t + k$

becomes:

$\ln r(0) = (\lambda + i\omega)0 + k$

$\ln r(0) = 0 + k = k$

So

$$\ln r = [(\lambda + i\omega)t] + \ln r(0);$$

now  $\ln e = 1$  so can write:

$$\ln r = [(\lambda + i\omega)t] \ln e + \ln r(0);$$

$$\ln e = 1 = \ln \exp$$

where  $\exp = e$

Thus we have:

$$\ln r = \ln r(0) \exp(\lambda + i\omega)t$$

Take the  $\ln$  out you get

$$r = r(0) \exp(\lambda + i\omega)t$$

This solution is a translation [multiplying] and a rotation of  $r(0)$

If the orbit is the same of Earth around the Sun then

$$\lambda = 0 = \text{constant}$$

$$\text{Then } r = r(0) \exp[i\omega t]$$

Where  $\exp[i\omega t] = \cos \omega t + i \sin \omega t$

$r(0)$  is the orbit at  $\theta = \text{zero}$  or any initial condition

you can take any point on the entire ellipse as an initial condition and for an arbitrary

condition it is equal to the original solution of Newton which is

$$r(0) = r(\theta, 0) = a(1 - \epsilon^2)/(1 + \epsilon \cos \theta) \text{ no time}$$

For the long explanation of this equation, see Appendix.

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What he means by “no time” is what he calls “Newtonian time independent equation”

While, when he says:

$r = r(0) \exp[i\omega t]$  = time dependent solution

He calls this the “Newtonian time dependent equation.”

Joe says:

This Newton’s time dependent solution has been missed by academia.

Newton's time dependent solution is a wave solution; it is what quantum mechanics uses.

You can take classical Newtonian mechanics (of the  $r = r(0)$ ) and multiply it with  $\text{Exp}[i\omega t]$  and get quantum mechanics, but it is all really from Newtonian physics.

This is quite amazing; the same sort of equation as Quantum mechanics works by is derived by Newtonian physics. But in the context of Newtonian physics this equation would be interpreted by a Newtonian perspective. While from Quantum mechanics it would be interpreted by Quantum mechanic beliefs. Since the same sort of equation can be derived from Newtonian perspective there is no real need to switch to a Quantum perspective (a Quantum perspective as created by the philosophy of the Copenhagen interpretation). We could have stated with Newtonian physics in the 1920-30s instead of having the supposed Quantum revolution.

The issue then becomes how this could have happened.

And that is answered by Joe as “they” did not deal with Newtonian physics correctly, he says:

350 years ago they did not solve the equation correctly.

The reason why they missed the time dependent solution back then, Joe thinks because back then the equipment they used were rough, there was no modern lighting equipment to work at night, and the static solution (time independent) was sufficient for their needs without the need for the more sophisticated time dependent solution. Newton was told that earth motion around the sun was an ellipse so he found the solution for the ellipse – which was the static solution. No one thought about a rotating ellipse. If someone said it is a rotating ellipse probably someone would have then derived the time dependent solution. If you want to rotate then you need the solution with the  $\text{Exp}[i\omega t]$ . Anyway, the time dependent solution is not in Newton's literature

So “they” only derived the Newtonian time independent solution and did not derive the Newtonian time dependent solution. There were two solutions, they only spotted the one solution and missed the other solution.

When the second solution was discovered instead of recognizing it as still being part of Newtonian physics, they called it Quantum mechanics. And then “they” reinterpreted that second solution from a non-Newtonian perspective that “they” called Copenhagen interpretation.

This is quite shocking – “they” do their maths wrong, and instead of correcting their mistakes, “they” bury those mistakes in more mistakes.

If we now try to connect the two theories, we might try to explain it thus:

If we call the time independent solution as Newton Physics, then Newton's time dependent equation is the product of two quantities let us called “Newton quantity” and “quantum quantity” then we have:

Newton quantity X quantum quantity

where quantum quantity is  $\exp[i\omega t]$  in the general case

but when quantum quantity is 1 then you see Newtonian Physics (time independent).

According to Joe - the subtraction of the two quantities is relativistic.

As Joe explains it – the time dependent Newtonian solution explains it all. It explains:

1. Wave-particle duality

The time independent equation is expressing a particle or planet movement in an ellipse

The time dependent equation is expressing a wave motion of the same particle with amplitude equals to the time independent part.

2. It combines classical [particle] with Quantum[wave]

3. The difference between the two equations is "relativistic effects"

Classical quantum relativity all in one equation

## **References**

[1] Einstein's Nemesis # 1: DI Herculis Binary Stars Apsidal Motion Solution, Joe Nahhas <http://www.wbabin.net/physics/nahhas.pdf>

[2] Einstein Nemesis # 2: As Camelpardis Apsidal Motion Puzzle Solution, Joe Nahhas  
<http://www.wbabin.net/physics/nahhas2.pdf>

[3] Relativity Theory = Visual deceptions  $S = r \exp \_ \_ t$ ;  $\sin \_ t = v/c$ . Joe Nahhas  
<http://www.wbabin.net/physics/nahhas3.pdf>

[4] Planet Mercury: Advance of Perihelion of 43" Seconds of an Arc per Century, Joe Nahhas <http://www.wbabin.net/physics/nahhas4.pdf>

[5] Areal velocity, Answers.com <http://www.answers.com/topic/areal-velocity>

[6] University Physics Hugh D Young, Roger A Freedman, Addison-Wesley Publishing, 1996 ISBN 0-201-31132-1 p 370

[7] There are dozens of references on the internet that explain this fact e.g. Orbital Energies, Kepler's Laws and Other Relationships, by Larry Bogan - <http://www.go.ednet.ns.ca/~larry/orbits/kepler.html> quote: "Kepler discovered first that planets move in elliptical orbits about the Sun. The 2nd Law of Kepler describes the relative velocity of the objects in their elliptical orbits. He discovered that the line from the Sun to the planet swept out equal areas in equal times."

## **Addendum**

The most crucial insight seems to come with the equation:

$$2(r'/r) = -(\theta''/\theta')$$

Nahhas takes this as equal to a complex number.

This equation is normally taken as equal to a real number.

i.e. Newtonian physics was based on using real numbers; but when complex numbers were discovered; they forgot to update Newtonian physics with using complex numbers.

Newton's theory was of wave-particle duality of light; but that got modified by later Newtonians as just a particle theory of light. (see another of my papers) Updating Newtonian physics with complex numbers gives the wavefunction; hence we have wave-particle duality as Newton originally intended. Indeed Schrodinger's equation was built from Newtonian physics and then had the wavefunction added. Newtonian physics and Quantum mechanics are more closely connected that the Establishment cares to admit to.

Joe Nahhas is looking at these things with a lot less prejudice than physicists are taught to believe, and getting to the more fundamental nature of things.

Taking the left-hand side of the equation

$$2(r'/r)$$

he is setting that as equal to a complex number, and does the fiddle with the 2 to lose it in the complex number:

$$(r'/r) = \text{complex number}$$

which then can be written:

$$(1/r) (dr/dt) = \text{complex number}$$

move the dt to the other side and integrate:

$$\text{integral } (1/r)dr = \text{integral } (\text{complex number})dt$$

It is quite amazing that Nahhas insight has not been noticed before; but physicists must have just got locked into a fixed mind-set that they were dealing with real number system and did not realize that the more general solution for the Newtonian equation was in the complex number system. (If we wanted to - of course we could extend to other number systems such as hyper-complex numbers; but for the time being complex numbers suffice for the link of Newtonian Physics with modern day physical observations.)

c.RJAnderton2009

\* Update for reference to extra article by Joe Nahhas 2009-03-24

\*\* update for addendum 2009-05-02 and see \*\* in text

**Appendix: Long explanation of equation used in text**

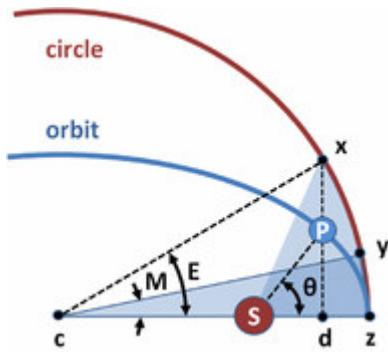


Figure : Geometric construction for Kepler's calculation of  $\theta$ . The sun (located at the focus) is labeled  $S$  and the planet  $P$ . The auxiliary circle is an aid to calculation. Line  $xd$  is perpendicular to the base and through the planet  $P$ . The shaded sectors are arranged to have equal areas by positioning of point  $y$ .

The Keplerian problem assumes an elliptical orbit and the four points:

- $s$  the sun (at one focus of ellipse);
- $z$  the perihelion
- $c$  the center of the ellipse
- $p$  the planet

and

- $a = |cz|$  distance between center and perihelion, the **semimajor axis**,
- $\varepsilon = \frac{|cs|}{a}$ , the **eccentricity**,
- $b = a\sqrt{1 - \varepsilon^2}$  the **semiminor axis**,
- $r = |sp|$  the distance between sun and planet.
- $\theta = \angle zsp$  the direction to the planet as seen from the sun, the **true anomaly**.

The problem is to compute the polar coordinates  $(r, \theta)$  of the planet from the **time since perihelion**,  $t$ .

It is solved in steps. Kepler considered the circle with the major axis as a diameter, and

- $x$  the projection of the planet to the auxiliary circle
- $y$  the point on the circle such that the sector areas  $|zcy|$  and  $|zsx|$  are equal,
- $M = \angle zcy$  the **mean anomaly**.

The sector areas are related by  $|zsp| = \frac{b}{a} \cdot |zsx|$ .

The circular sector area  $|zcy| = \frac{a^2 M}{2}$ .

The area swept since perihelion,

$$|zsp| = \frac{b}{a} \cdot |zsx| = \frac{b}{a} \cdot |zcy| = \frac{b}{a} \cdot \frac{a^2 M}{2} = \frac{abM}{2},$$

is by Kepler's second law proportional to time since perihelion. So the mean anomaly,  $M$ , is proportional to time since perihelion,  $t$ .

$$M = \frac{2\pi t}{P},$$

where  $P$  is the orbital period.

The mean anomaly  $M$  is first computed. The goal is to compute the true anomaly  $\theta$ . The function  $\theta=f(M)$  is, however, not elementary. Kepler's solution is to use

$$E = \angle zcx, x \text{ as seen from the centre, the } \mathbf{eccentric\ anomaly}$$

as an intermediate variable, and first compute  $E$  as a function of  $M$  by solving Kepler's equation below, and then compute the true anomaly  $\theta$  from the eccentric anomaly  $E$ . Here are the details.

$$\frac{|zcy|}{\frac{a^2 M}{2}} = \frac{|zsx|}{\frac{a^2 E}{2}} = \frac{|zcx| - |scx|}{\frac{a\varepsilon \cdot a \sin E}{2}}$$

Division by  $a^2/2$  gives **Kepler's equation**

$$M = E - \varepsilon \cdot \sin E.$$

The catch is that Kepler's equation cannot be rearranged to isolate  $E$ . The function  $E = f(M)$  is not an elementary formula, but Kepler's equation is solved either iteratively by a root-finding algorithm or, as derived in the article on eccentric anomaly, by an infinite series.

Having computed the eccentric anomaly  $E$  from Kepler's equation, the next step is to calculate the true anomaly  $\theta$  from the eccentric anomaly  $E$ .

Note from the figure that

$$\vec{cd} = \vec{cs} + \vec{sd}$$

so that

$$a \cdot \cos E = a \cdot \varepsilon + r \cdot \cos \theta.$$

Dividing by  $a$  and inserting from Kepler's first law

$$\frac{r}{a} = \frac{1 - \varepsilon^2}{1 + \varepsilon \cdot \cos \theta}$$

to get

$$\begin{aligned} \cos E &= \varepsilon + \frac{1 - \varepsilon^2}{1 + \varepsilon \cdot \cos \theta} \cdot \cos \theta \\ &= \frac{\varepsilon \cdot (1 + \varepsilon \cdot \cos \theta) + (1 - \varepsilon^2) \cdot \cos \theta}{1 + \varepsilon \cdot \cos \theta} = \frac{\varepsilon + \cos \theta}{1 + \varepsilon \cdot \cos \theta}. \end{aligned}$$

The result is a usable relationship between the eccentric anomaly  $E$  and the true anomaly  $\theta$ .

A computationally more convenient form follows by substituting into the trigonometric identity:

$$\tan^2 \frac{x}{2} = \frac{1 - \cos x}{1 + \cos x}.$$

Get

$$\begin{aligned} \tan^2 \frac{E}{2} &= \frac{1 - \cos E}{1 + \cos E} = \frac{1 - \frac{\varepsilon + \cos \theta}{1 + \varepsilon \cdot \cos \theta}}{1 + \frac{\varepsilon + \cos \theta}{1 + \varepsilon \cdot \cos \theta}} \\ &= \frac{(1 + \varepsilon \cdot \cos \theta) - (\varepsilon + \cos \theta)}{(1 + \varepsilon \cdot \cos \theta) + (\varepsilon + \cos \theta)} \\ &= \frac{1 - \varepsilon}{1 + \varepsilon} \cdot \frac{1 - \cos \theta}{1 + \cos \theta} = \frac{1 - \varepsilon}{1 + \varepsilon} \cdot \tan^2 \frac{\theta}{2}. \end{aligned}$$

Multiplying by  $(1+\varepsilon)/(1-\varepsilon)$  and taking the square root gives the result

$$\tan \frac{\theta}{2} = \sqrt{\frac{1 + \varepsilon}{1 - \varepsilon}} \cdot \tan \frac{E}{2}.$$

We have now completed the third step in the connection between time and position in the orbit.

One could even develop a series computing  $\theta$  directly from  $M$ .

The fourth step is to compute the heliocentric distance  $r$  from the true anomaly  $\theta$  by Kepler's first law:

$$r = a \cdot \frac{1 - \varepsilon^2}{1 + \varepsilon \cdot \cos \theta}.$$

language of Kepler's laws applies as the complicated orbits are described as simple Kepler orbits with slowly varying orbital elements. See also Kepler problem in general relativity.

While Kepler's laws are expressed either in geometrical language, or as equations connecting the coordinates of the planet and the time variable with the orbital elements, Newton's second law is a differential equation. So the derivations below involve the art of solving differential equations. Kepler's second law is derived first, as the derivation of the first law depends on the derivation of the second law. The derivations that follow use heliocentric polar coordinates, that is, polar coordinates with the sun as the origin.

From wikipedia 12 march2009

Ref: [http://en.wikipedia.org/wiki/Kepler's\\_laws\\_of\\_planetary\\_motion\\_12\\_march2009](http://en.wikipedia.org/wiki/Kepler's_laws_of_planetary_motion_12_march2009)

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