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Resolution of the Ehrenfest Paradox

Jaroslav Hynecek[1]

ABSTRACT

In this article a possible resolution of the famous Ehrenfest paradox [1] is offered. The paradox relates to a spinning disc and the Special Relativity Theory (SRT) applied to it. The resolution of the paradox is based on the proposition that the paradox results from an incorrect application of SRT to a system that is not in inertial motion. The centrifugal and centripetal forces resulting from the rotation are always present and need to be included into considerations. Using the previously derived metric for a centrally gravitating body the effect of the centrifugal and centripetal forces can be included. When this is correctly done no paradox is obtained and it is shown that the spinning disc has flat space-time geometry. The measured data from the experiments conducted on such rotating systems are explained by inertial mass increase as described by SRT.

Key words: Ehrenfest paradox, Mosbauer effect, Special Relativity Theory. General Relativity Theory, Schwarzschild metric, New Space-Time Metric.

INTRODUCTION

There have been many papers published on the resolution of the Ehrenfest paradox with various degrees of success and conclusions [1]. The paradox results from considering the application of Lorentz coordinate transformation from SRT to a spinning plate. The plate circumference should contract while the radius should not since the motion of the radius is always perpendicular to the plate's rotating direction. Thus the circumference is no longer equal to $L_o = 2\pi \cdot R$. This leads to a non-flat geometry, which is not a domain of SRT. From this consideration it is clear that only the kinematic approach to this problem, as offered by SRT, is not enough. SRT deals with the systems in inertial motion and does not account for the gravitational or inertial forces. In order to resolve the paradox, it is necessary to use a metric from GRT that describes a non-flat space-time geometry that may be adopted to include the centrifugal and centripetal forces in the system. The well known metric describing the space-time around a

1 E-mail: jhynecek@netscape.net,

centrally gravitating body that has a mass M is the Schwarzschild metric:

$$ds^2 = (1 - R_s / r)c^2 dt^2 - (1 - R_s / r)^{-1} dr^2 - r^2 \left(d\vartheta^2 + \sin\vartheta^2 d\varphi^2\right). \qquad (1)$$

Here $R_s = 2\kappa M / c^2$ is the Schwarzschild radius, κ the gravitational constant, and c the speed of light. However, new metric has been recently published, based on the minimum of energy stored in the gravitational filed [2], which is believed to be more accurate and more closely describes the reality:

$$ds^2 = e^{\frac{-R_s}{\rho}} c^2 dt^2 - e^{\frac{R_s}{\rho}} dr^2 - \rho^2 e^{\frac{-R_s}{\rho}} \left(d\vartheta^2 + \sin\vartheta^2 d\varphi^2\right). \qquad (2)$$

The parameter ρ in this metric is defined by a differential equation as follows:

$$d\rho = e^{\frac{R_s}{2\rho}} dr. \qquad (3)$$

The new metric can now be used to resolve the paradox. In order to find the solution it is possible to generalize the metric in Eq.2 as follows:

$$ds^2 = e^{\frac{2\phi}{c^2}} (cdt)^2 - d\rho^2 - \rho^2 e^{\frac{2\phi}{c^2}} \left(d\vartheta^2 + \sin^2\vartheta \cdot d\varphi^2\right), \qquad (4)$$

where the term -Rs/ρ was replaced by the corresponding generalized gravitational-like potential. An observer placed on the spinning plate circumference and rotating with it observes a centrifugal force. This force is balanced by the mechanical centripetal force of the plate to keep the system in equilibrium. To solve the paradox the problem can be divided into two steps: first, the centripetal force of the disc acting to counteract the centrifugal inertial force can be simulated for small velocities locally at radius R ≈ ρ by a non rotating special gravitational-like field that is pointing inward to the center of the plate. The simulated gravitational-like potential, which can then be substituted into Eq.4 can have a classical from expressed in terms of the circumference velocity: $\phi_n = \frac{1}{2} v^2$. The force resulting from this potential thus completely images the centrifugal force and fully compensates its effects. In the second step, by considering a stationary observer relative to the plate and thus not considering any Coriolis acceleration and also considering that this observer now does not feel any radial acceleration, the standard SRT Lorentz transform can be used for the coordinate transformation from the rotating to the laboratory system. The metric line element for the curved space-time approximating the centripetal force by the fictitious gravitational-like force is thus as follows:

$$ds^2 = e^{\frac{v^2}{c^2}} (cdt)^2 - dR^2 - R^2 e^{\frac{v^2}{c^2}} d\varphi^2. \qquad (5)$$

By expanding the exponential terms into the power series it is easily seen that the circumference length as observed by the laboratory observer is:

$$L_i \approx L_o\left(1+\frac{1}{2}\frac{v^2}{c^2}\right), \tag{6}$$

while due to the Lorentz contraction the circumference length is:

$$L_c \approx L_o\left(1-\frac{1}{2}\frac{v^2}{c^2}\right). \tag{7}$$

The effects thus cancel each other to the second order of v/c and no paradox results. The laboratory observer will thus see the disk periphery not contracted. The similar conclusion is obtained from the metric of Eq.5 also for time. The simulated centripetal force causes the time contraction while the Lorentz time dilation compensates this effect. The space-time geometry of the rotating disc as viewed by the laboratory observer is thus approximately flat. It thus seems that all the SRT effects are being compensated by the curved metric and the only remaining SRT effect that is not compensated for is the inertial mass increase. This also lends validity to the classical explanation and the classical formula for the Sagnac effect without any need for SRT theory.

It is interesting to note that the standard Schwarzschild metric does not offer similar solution to the Ehrenfest paradox. This results from the incorrect metric coefficient standing by the angular coordinates. The solution of the Ehrenfest paradox using the new metric thus provides additional support for its correctness.

There have been many experiments performed in the past in rotating systems to confirm various GRT phenomena, but as is clear from the above explanation only the SRT inertial mass increase, and the effects related to the inertial mass increase such as the absorption line shift in the Mossbauer Fe^{57} effect can be observed [3]. No GRT effects can be measured in these experiments to the second order of v/c. It is also necessary to understand in detail the construction of the clock to make sure that the time is what is measured by it not the inertial mass increase.

CONCLUSIONS

The new space-time metric is used to suggest another resolution of the Ehrenfest paradox and the related experimental verifications. This further validates the metric correctness.

REFERENCES

1. **http://en.wikipedia.org/wiki/Ehrenfest_paradox.**
2. J. Hynecek, "Remarks on the Equivalence of Inertial and Gravitational Masses and on the Accuracy of Einstein's Theory of Gravity", Physics Essays, v 18, No. 2, Jun 2005, p182.
3. H.J. Hay, J.P. Schiffer, T.E. Cranshaw, P.A. Egelstaff, Physical Review Letters, vol. 4, Issue 4, pp. 165-166, Feb.1960.