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On the Sagnac Effect

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Introduction

Let b be a source of light stationary in frame s and moving with velocity u relative to frame S, and let B be its position in S when a pulse of light is emitted from b. Let X be the distance of b from an observer O in S when light is emitted. Assume that o is an s observer that is contiguous to O when light is received. It was shown in [1-3, www.wbabin.net] that all observers (the stationary and the moving observers) agree on assigning the same distance x and time intervals t with the light trip $(b \to o \; or \; O)$, and that

(1) $$x = \sqrt{\frac{1-\beta}{1-\beta}} X = \Gamma^{-1} X \qquad t = \sqrt{\frac{1-\beta}{1+\beta}} T \qquad (with \, t = x/c)$$

if b is approaching O, and

(2) $$x = \Gamma X, \qquad t = \Gamma T$$

if b recedes from O.

The Sagnac Effect

We shall be contented with checking the performance of the contraction-elongation relations (1) and (2) in dealing with Sagnac effect corresponding to electromagnetic transmission, where spinning of the earth about its axis amounts for a complete round to approximately 207 ns advance (delay) for a wave propagating westward (eastward) parallel to the earth's equator [4].

Let S be the inertial frame in which the earth is spinning about its axis with an angular frequency w. Relative to S the velocity of each point of the earth's equator is $u = wR_e$, where R_e is the earth radius at its equator. Assume that two pulses of light are emitted simultaneously from a point o at the equator in opposite directions in parallel to the equator. Had the earth been stationary, the S observers (now they include these attached to the earth) find that the time taken by a pulse of light to travel round the earth given by $T = circumference/c = 40{,}000/300{,}000 = 2/15\,s$. Let $X_w(X_e)$ be an S observers that is contiguous to o when light emitted from o westward (eastward) is received back by o and hence by $X_w(X_e)$. The emitter o is approaching X_w during the light trip $(o \to X_w)$ and receding from X_e during the light trip $(o \to X_e)$. According to the content of the introduction, all observers (including these attached to the earth and hence rotating with it) attribute to the trip

$(o \to X_w)$ the period $t_w = T/\Gamma$, and to the trip $(o \to X_e)$ the period $t_e = \Gamma T$. It follows that the wave emitted westward returns to *o* advancing with the time interval

(3) $$T - t_w = T - T/\Gamma = T(1 - 1/\Gamma) \approx 206.36 \text{ ns.}$$

And the wave emitted eastward comes back to o with a delay

(4) $$t_e - T = \Gamma T - T = T(\Gamma - 1) \approx 206.36 \text{ ns.}$$

The linear velocity of the earth at its equator, appearing in $\Gamma(u)$, was taken when calculating (3) and (4)

(5) $$u = 40{,}000/23.56 \times 60 \times 60 \approx 0.4643\, km/s$$

The following quantity

(6) $$t_e - t_w = \Gamma T - \Gamma^{-1} T = \frac{2(u/c)T}{\sqrt{1 - u^2/c^2}} \approx 2 \times 206.36 ns$$

represents the delay period between receiving the pulse emitted westward from that emitted eastward by the observer *o*.

It may be worth to mention that the contraction-elongation formula can be linked with the GLT [5] as follows: Define the so-called scaling factor by

(6a) $$\Gamma(u,\theta) = \frac{\sqrt{1 - \beta^2 \sin^2\theta} + \beta\cos\theta}{\sqrt{1 - \beta^2}},$$

Then the GLT [5] reduces for $r = ct$ to

(7) $$R = \Gamma(u,\theta) r, \quad R = cT, \quad ,\phi = \phi', \quad \theta = \theta'$$

The last equation (7) reduces to (2) for $\theta = 0$ and to (1) for $\theta = \pi$.

References

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