The Scaling Theory IX: Explaining the Non-Null Results of Michelson and Morley Experiment

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15. The Non-Null Results of the Michelson and Morley Experiment

15.1. The Michelson's and Morley's Experiment

The well-known Michelson's and Morley's experiment (MM for short) was designed to detect the earth's motion through the ether by measuring the time difference between the durations of two return light's trips along two perpendicular arms of an interferometer, with one arm points in the direction of the almost translational motion of the earth around the sun and the other is perpendicular to it. Both trips start from the same point *o*, make paths perpendicular to each other, and after covering the same distance *L* along the perpendicular axes of the interferometer, return to their starting point *o*.

The observed effect was much less than expected. Similar experiments were carried out by other scientists[30,15], and the same result was found: the observed fringe shift is much less than the calculated one.

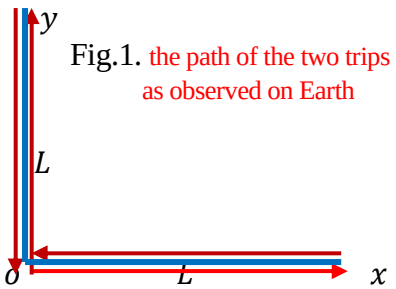


Fig.1. the path of the two trips as observed on Earth

Fig.1. The blue lines represent the interferometer's arms

Although the results of MM were described as null, they weren't really so[26]. In this article we targets two objectives

(1) We argue that the expected effect in the MM and similar experiments is due to the rotational motion of the earth about its axis, but not to its orbital motion around the sun. This argument is based on the following facts:

- There is no ether in the scaling theory.
- The time we use on earth is the time of the timed frame *S* translating with the earth in its orbital motion, but not rotating relative to fixed stars. Hence there can be no fringe displacement due to the earth's orbital motion.
- A frame *s* with origin *o* at the *earth surface* and rotating with the earth can be considered during a short period of time an inertial frame that is translating relative to *S* at the linear velocity of its origin *o*, which is the rotational velocity of the earth surface at *o*.

(2) We show that a deviation of the perpendicular interferometer's arms, accidental or designed, by an unspecified minute amount from a right angle can produce a somehow chaotic observed non-null effect.

In the following treatment, it is assumed that the reader is well informed of the MM experiment which can be found in most text books on special theory of relativity[15,30] .

The frame *S*, with origin *O* at the earth's center, which does not rotate relative to distant stars can be considered inertial, for it executes within a small period of time only a translational motion. Relative to *S* the earth spins about its axis with a constant angular velocity, and the linear velocity of a point *o* of the earth's surface is $u\ sin\theta$, where *u* is the linear velocity of a point of the earth's equator, and θ is the co-latitude of the point *o*. For simplicity we assume temporarily that the experiment is carried out at the earth equator, with one of the arms pointing eastwards and the other northwards. Let $s \equiv oxy$ be a frame rotating with the earth, with origin at the light's source *o*, the *x*-axis points eastwards, and *oy* northwards. In the MM experiment two light's beams are sent simultaneously from *o* towards two mirrors on the perpendicular arms at the same distance *L* from *o*, where they are reflected back to *o*. Since both trips are bound in *s*, i.e. their initial and final ends are determined from start, the bound scaling transformations are applicable. We shall call the 2-way trip taking place on the *x*-axis the return longitudinal, or just the longitudinal, trip, and call the other 2-way trip the traverse trip. In the longitudinal trip, the rotational velocity of the earth's surface and the velocity of the light's beams are either parallel or anti-parallel, whereas these velocities are perpendicular in the traverse trip.

The following two figures, which are not necessary in calculations, illustrate the paths of the two trips as seen in the timed inertial frame *S*.

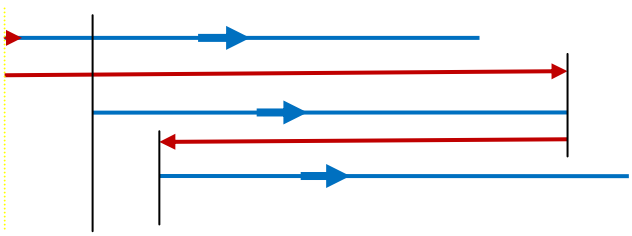


Fig. 2. The longitudinal return trip as observed in S.
The blue lines represent the longitudinal arm. The blue vectors represent the velocity of *s* relative to *S*

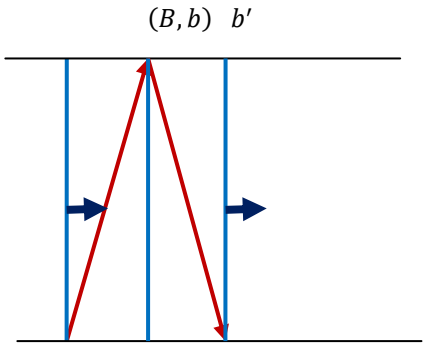


Fig.3. The traverse return trip as observed in S.

Using the frame *s'* view (see section 13.3), the length of the return longitudinal trip is

$$l_{long} = l_+ + l_- = [\Gamma(\beta, 0) + \Gamma(\beta, \pi)]L$$

(15.1) $$= L\sqrt{\frac{1+\beta}{1-\beta}} + L\sqrt{\frac{1-\beta}{1+\beta}} = \frac{2L}{\sqrt{1-\beta^2}},$$

and the length of the return traverse trip is

(15.2) $$l_{trav} = l_{1trav} + l_{2trav} = 2\Gamma\left(\beta, \frac{\pi}{2}\right)L = 2L.$$

The difference between the lengths of the light's paths is

(15.3) $$\Delta = l_{long} - l_{trav} = 2L\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right)$$

$$\approx 2L\left(\frac{1}{2}\beta^2\right) \approx L\left(\frac{u}{c}\right)^2 meter,$$

which corresponds to $\frac{1}{2}f = (\frac{L}{\lambda})\beta^2$ fringes. When the interferometer is rotated by a right angle the difference doubles giving rise to a fringe shift

(15.4) $$f = \frac{2\Delta}{\lambda} = \frac{2L}{\lambda}(\frac{u}{c})^2,$$

where λ is the wavelength of the used light. In the MM (1987) experiment

$$\lambda = 6 \times 10^{-7} meter, L = 1100\ cm.$$

Substituting $u = 0.46425 km/s$ and $c = 298792.5 km/s$ we obtain

(15.5) $$f \approx 0.0001,$$

which is just (1/4000) of the commonly predicted value, and (1/100) of the observed result, which is 0.01 fringe. It noted that the scaling theory yields the fraction

(15.6) $$(v_{rot}/v_{tra})^2 = (0.464/30)^2 \approx 0.00024$$

of the predicted shift in MM similar experiments. In other wards,

$$f_{cal(scal)}/f_{pre} \approx 0.00024.$$

Going through the results of various trials of the MM experiments listed in Wikipedia[30], or in French[15], we note that the observed effect corresponds to a velocity *u* that is always greater than the rotational speed of the earth about its axis. According to the Wikipedia, the least upper limit of *u*, which is therein the orbital velocity of the earth (30km/sec), while in our arguments it is the linear rotational speed of the earth's surface at the equator (0.464km/sec), occurs in the Illingworth's trial (1927) with *u* is less than one kilometer per second. The second reasonable upper limit is found in Joos' trial (1930), with $u = 1.55\ km/s$. In fact the MM experiment yields a fringe shift corresponding to an orbital speed $u = 4.75\ km/s$, which is still closer to the value of the linear rotational speed, $0.46425\ sin\theta\ km/s$, of the location at which the experiment is carried out.

15.2. Deviation from Right Angle

Although the results of the MM experiment and similar versions were reported as null, they were not really so. The following table shows the ratio f_{obs}/f_{exp} between the number of observed fringes to that calculated using the ether model for some versions of MM experiment.

Experiment Date	MM 1881	MM 1887	Miller 1921	Miller 1926	Tomasckeck 1924	Picard 1927
	0.5	0.025	0.067	0.077	0.067	0.05

We investigate here the effect of deviation of the rays from the arm's directions. We shall confine our study to the case of east-west and north-south trips, or as to say, to longitudinal and traverse trips. Any other two perpendicular directions are analysed in a similar way.

For $u \ll c$, we have

$$\Gamma(\beta,\theta) \approx \left\{\beta cos\theta + \left(1 - \frac{1}{2}\beta^2 sin^2\theta\right)\right\}\left(1 + \tfrac{1}{2}\beta^2\right)$$

(15.7) $$\approx 1 + \beta cos\theta + \tfrac{1}{2}\beta^2 cos^2\theta,$$

(15.8) $$\Gamma(-\beta,\theta) \approx 1 - \beta cos\theta + \tfrac{1}{2}\beta^2 cos^2\theta.$$

For $\theta = \frac{\pi}{2} + \alpha$, where α is sufficiently small, we have

$$cos\theta = -sin\alpha = -\alpha + \frac{1}{3!}\alpha^3 \ldots \approx -\alpha$$

$$sin\theta = cos\alpha \approx 1 - \frac{1}{2}\alpha^2$$

Thus for $\beta \ll 1$ and $|\alpha| \ll 1$, we have for $\theta = \frac{\pi}{2} + \alpha$

(15.9) $$\Gamma\left(\beta, \frac{\pi}{2} + \alpha\right) \approx 1 - \alpha\beta + \tfrac{1}{2}\alpha^2\beta^2 \approx 1 - \alpha\beta$$

For $\theta = \alpha$, where α is sufficiently small ,i.e. in the vicinity of $\theta = 0$, we have

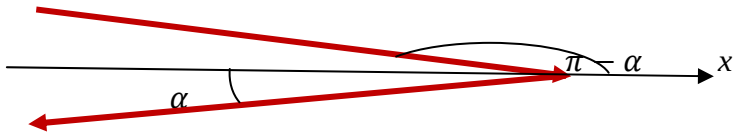
$$\Gamma(\beta,\alpha) \approx \left\{\beta(1 - \frac{1}{2}\alpha^2) + \left(1 - \frac{1}{2}\beta^2\alpha^2\right)\right\}\left(1 + \tfrac{1}{2}\beta^2\right)$$

(15.10) $$\approx 1 + \beta - \frac{1}{2}\beta\alpha^2 + \frac{1}{2}\beta^2(1 - \alpha^2),$$

(15.11) $$\Gamma(\beta,\pi - \alpha) \approx 1 - \beta + \frac{1}{2}\beta\alpha^2 + \frac{1}{2}\beta^2(1 - \alpha^2).$$

The length of the two way trip along *ox*, i.e, the length of the longitudinal east-west trip is

(15.12) $$\approx [\Gamma(\beta,\alpha) + \Gamma(\beta,\pi - \alpha)]L = 2L[1 + \frac{1}{2}\beta^2(1 - \alpha^2)] \approx 2L[1 + \frac{1}{2}\beta^2]$$



This shows that the length of the two way longitudinal trip is stable to the first order under minute deviation α from an exactly longitudinal directions (i.e. directions along and opposite to *ox*). The length of the traverse 2-way trip along *oy*, i.e. the north-south trip, is

(15.13) $$\approx 2L.\Gamma\left(\beta, \frac{\pi}{2} + \alpha\right) = 2L(1 - \alpha\beta),$$

which shows that the effect of a minute deviation α from traverse direction produces an increment $-\alpha\beta(2L)$ in the length of the traverse return trip, which otherwise would be of length *2L*. This increment is proportional to the deviation, to the original length of the traverse trip, and to β. The difference between the altered lengths of the longitudinal and traverse trips is

(15.14) $$\Delta_1 = 2L\left(1+\tfrac{1}{2}\beta^2\right) - 2L(1-\alpha\beta) = L\beta^2 + 2L\alpha\beta.$$

Employing equation (15.3) which gives the difference in path length between the longitudinal and traverse trips without doubling this effect we write the latter equation in the form

(15.15) $$\Delta_1 = \Delta + \frac{\alpha}{\beta}\Delta.$$

A rotation through a right angle doubles this difference, giving rise to a difference

(15.16) $$\Delta' = 2\Delta_1 = 2\Delta\left(1+\frac{\alpha}{\beta}\right),$$

which corresponds to a number of fringes f' given by

(15.17) $$f' = \frac{\Delta'}{\lambda} = \frac{2\Delta}{\lambda}\left(1+\frac{\alpha}{\beta}\right) = f\left(1+\frac{\alpha}{\beta}\right).$$

Thus when the beams of light are made to deviate from right angle by a minute amount α, intentionally or as a result of an experimental inaccuracy, the ratio of the resulting number of fringes f' to its number f associated with no deviation is

(15.18) $$\frac{f'}{f} = 1 + \alpha\frac{c}{u}.$$

Solving for the deviation we get

(15.19) $$\alpha = \left(\frac{f'}{f} - 1\right)\frac{u}{c},$$

in which $f'$is number of fringes intimate to a minute deviation α , and f is the number of fringes when no deviation exist.

The Deviation could be due to

- Casual reasons such as seismic vibrations, human made shock waves, or experimental errors.
- Intentional reasons in which the path of light is lengthened through folding it on itself by a series of mirrors.

In the MM experiment (1887), for example, deviation is necessary to reflect light, back and forth, in a set of mirrors in aim to lengthen its path. The scaling theory predicts for MM experiment a fringe shift $f = 0.0001$, and in order to obtain $f' = 0.01$ as it is observed, we should make a deviation given by (15.19), which is in this case

$$|\alpha| = \left(\frac{0.01}{0.0001} - 1\right)\frac{0.464}{300{,}000} \approx 1.5312 \times 10^{-4} radian.$$

Thus a deviation of the traverse 2-way trip by 32 $arcsec$ from a right angle will produce 0.01 fringe as was obtained in (1887) MM experiment.

According to the sketch of the MM interferometer given in French (page 52), and due to consecutive reflections in adjacent mirrors, it seems to me that the rays employed in the experiment did deviate from right angle. Moreover, the general pattern of results of various experiments similar to that of MM's (Wikipedia[30], French[15](page 53)) shows that when the arms' length is increased the fringe shift decreases. This fact may be attributed to a more accuracy in achieving a right angled arms because of their greater length.

Using equation (15.6), the scaling theory predicts in correspondence with the calculated shift in Joos experiment (1930) which is $f_{exp} = 0.75$ fringes, the shift

$$f_{cal(scal)} \approx 0.00018\, fringes,$$

which is about ten times less than the observed one ($f'_{obs} = 0.002$). The deviation required to produce this observed result is

$$|\alpha| = \left(\frac{f'_{obs}}{f_{cal(scal)}} - 1\right) = \left(\frac{0.002}{0.00018} - 1\right)\frac{0.464}{300{,}000} \approx 1.56 \times 10^{-5} radian$$

Thus a deviation from right angle by $|\alpha| \approx 0.00089°$ will produce the result obtained by Joos. In Miller's (1926) experiment, $f_{exp} = 1.12, f'_{obs} = 0.08$. The prediction of the scaling theory is $f_{cal(scal)} \approx 0.00027$, and the deviation required to produce this result is about $4.56 \times 10^{-4} rad$.