

Ballistic Doppler Beaming:

A Brief Investigation of the Headlight Effect and Aberration of Light

A. A. Faraj

a_a_faraj@hotmail.com

Abstract:

In the current investigation, the equations for calculating the headlight effect and light aberration, based upon the assumption of ballistic and relative speeds of light, have been derived. And the main characteristics of shifted electromagnetic radiation, due to these two optical effects, within the subluminal, near superluminal, and far superluminal regions, are analyzed and discussed in detail.

Keywords:

Headlight effect; light aberration; Doppler beaming; ballistic propagation of light; searchlight effect; the Galilean addition of velocities; emission theories of light.

Introduction:

Doppler beaming can be defined, within this context, as the bending of emitted and incident light rays in the forward direction, due to the motion of the emitting light source, the motion of the receiving observer, or both.

In the published literature, Doppler beaming of light, due to the motion of the emitting light source, is labeled as '*headlight effect*' and occasionally as '*searchlight effect*' [Ref. #5, #6, & #7].

While, almost always, Doppler beaming, due to the motion of the receiving observer, is labeled as '*light aberration*' [Ref. #1, #2, & #3].

And even though the headlight effect and light aberration can be observed only in the reference frame of the receiving observer, the headlight effect is much easier to visualize and to calculate in the reference frame of the emitting light source, in which the initial direction of emitted light is always observable.

Clearly, the headlight effect and, to a lesser degree, light aberration are among the most difficult optical phenomena to visualize and to compute on the assumption of independent velocity of light of the velocity of the light source. And none of the published investigations, in this regard so far, can be deemed as quite natural and convincing; let alone pleasantly elegant and appealing.

On the ballistic assumption of dependent velocity of light upon the velocity of the light source, however, the calculations, in the two cases, are relatively straightforward and simple:

- Compute the magnitude of the velocity resultant of light using the law of cosines.
- And insert the computed result into the standard equation of the law of sines in order to obtain the direction of light, in the case of headlight effect, and in the case of light aberration, respectively.

Nonetheless, on the ballistic assumption, the scope of the problems, under investigation, is much wider than that on the assumption of constant speed of light. And this is because, on the assumption of constant speed of light, only cases that satisfy the following condition have to be considered:

$$v < c$$

where v is the velocity of the light source or the velocity of the observer; and c is the muzzle speed of light.

By contrast, on the ballistic assumption of speed of light dependent upon the speed of the emitting light source, a comprehensive treatment of all cases within these three distinct regions is required:

1. The cases within the subluminal region as specified by the condition:

$$0 \leq v < c$$

2. The cases within the near superluminal region as defined by the condition:

$$c \leq v \leq 2c$$

3. And the cases in the far superluminal region defined by the condition:

$$2c < v < \infty$$

And that is because the main characteristics of shifted light, by the headlight effect and light aberration, in each one of the above regions, are significantly different in many respects.

For example, in the subluminal region, light emitted by the moving light source, in the backward direction along an angle of 180° , propagates outward at subluminal speeds, and shows necessarily Doppler red shifts whose numerical values increase linearly and in direct proportion with the speed of the light source.

But, in the near superluminal region, the same light emitted by the moving light source, in the backward direction, propagates inward in the direction of the moving light source, at subluminal speeds, and shows, as a result, reversed Doppler red shifts whose magnitudes decrease linearly and in inverse proportion with the speed of the light source in this region.

While, in the far superluminal region, the same light emitted by the moving light source, in the backward direction, propagates inward in the direction of the moving light source, at superluminal speeds, and exhibits, accordingly, reversed Doppler blue shifts whose magnitudes increase linearly and in direct proportion with the speed of the light source in the far superluminal region.

In any case, the main differences between the headlight effect and light aberration, on the basis of ballistic and emission theories, can be summarized in this brief list:

- The headlight effect can be caused only by the motion of the light source.
- Light aberration can be caused only by the motion of the observer.

- The headlight effect, in the subluminal region, does not alter the angular size of the light source at the time of reception.
- Light aberration, in the subluminal region, decreases the angular sizes for all light sources located, within a 2π -solid angle in the forward direction with respect to the velocity vector of the moving observer, at the time of reception. And at the same time, it increases the angular sizes of all light sources located, within a 2π -solid angle in the backward direction with respect to the velocity vector of the moving observer, at the time of reception.
- The headlight effect, in the subluminal region, neither shifts the direction nor changes the position of the light source at the time of emission.
- Light aberration, in the subluminal region, shifts the direction angle of incident light and changes the position of the light source at the time of emission.
- The headlight effect, in the near superluminal region, reverses the direction of propagation and the chronological order of events for light emitted along angles equal or very close to 180° in the backward direction with respect to the velocity vector of the moving light source.
- Light aberration, in the near superluminal region, reverses neither the direction of propagation nor the chronological order of events, in this case; because all incident light from stationary light sources, within a solid angle of 2π in the backward direction with respect to the velocity vector of the moving observer, simply, cannot be received by the fast moving observer.
- The headlight effect, in the far superluminal region, reverses the direction of propagation, the color of the Doppler shifts, and the chronological order of events for all light emitted within a solid angle of 2π in the backward direction with respect to the velocity vector of the moving light source.
- Light aberration, in the far superluminal region, reverses neither the direction of propagation, nor the color of the Doppler shifts, nor the chronological order of events, because, in this particular case, all incident light from stationary light sources, within a 2π -solid angle in the backward direction, can never be received by the fast moving observer.

1. The Doppler Beaming in the Subluminal Region:

As pointed out earlier, the subluminal region is defined by the following condition:

$$0 \leq v < c$$

where v is the velocity of the light source or the moving observer; and c is the muzzle speed of light.

And therefore, the speed of the emitting light source or the moving observer, v , in this region, does not exceed the canonical value of c .

In addition, the published literature, on Doppler beaming, is dedicated exclusively to the investigation of the headlight effect and light aberration in this specific region; and none is related to the near and far superluminal regions.

A. The Headlight Effect in the Subluminal Region:

By definition, the headlight effect is the bending of the light rays in the forward direction of the velocity vector of the emitting light source, as observed in the observer's frame of reference, relative to which the light source is in motion.

The headlight effect, and the Doppler effect as well, can be calculated, much more conveniently, in the the reference frame of the moving light source; but these two effects can only be observed and measured in the reference frame of the observer.

In the Reference Frame of the Moving Light Source:

Let v_s denote the velocity of the emitting light source with respect to the reference frame of the observer. And let ϕ denote the angle of emission, as measured in the reference frame of the emitting light source.

According to the Galilean addition of velocities, therefore, the velocity resultant of emitted light, c' , as computed in the reference frame, in which the emitting light source is at rest, can be obtained, in all cases, by using this general formula:

$$c' = \sqrt{c^2 + v_s^2 + 2cv_s \cos \phi} \quad 1.A.1$$

The direction of the velocity resultant, c' , is, within the framework of ballistic and emission theories, the precise quantitative definition of the headlight effect, ϕ' , which can be calculated by applying the law of sines to the arrangement under discussion:

$$\sin \phi' = \frac{c}{c'} \sin \phi = \frac{\sin \phi}{\sqrt{1 + \frac{v_s^2}{c^2} + 2 \frac{v_s}{c} \cos \phi}} \quad 1.A.2$$

where the angle ϕ is the initial direction of light emitted by a moving light source.

And consequently the Doppler effect, as calculated in the reference frame of the moving light source, is given by this equation:

$$f' = f \left(1 + \frac{v_s \cos \phi'}{c' - v_s \cos \phi'} \right) = f \left(1 + \frac{(v_s/c) \cos \phi'}{\sqrt{1 + \frac{v_s^2}{c^2} + 2 \frac{v_s}{c} \cos \phi} - (v_s/c) \cos \phi'} \right) \quad 1.A.3$$

where f is the emitted frequency; f' is the computed frequency; and Equation #1.A.3 is the standard equation for computing the Doppler effect in the reference frame of the moving light source.

Equation #1.A.2 gives the computed headlight effect; and Equation #1.A.3 produces the Doppler shifts, as calculated in the reference of the moving light source.

For example, if it's assumed that the velocity of the light source is:

$$v_s = 0.7c$$

then, the numerical values for the headlight effect and the Doppler effect can be computed by employing the above two equations for any number of given directions of emission, as shown in the following table:

<i>Initial Direction of Emission (ϕ)</i>	0.000 degrees	30.000 degrees	45.000 degrees	60.000 degrees	90.000 degrees	120.000 degrees	135.000 degrees	150.000 degrees	180.000 degrees
<i>Calculated Speed of Light (c')</i>	1.700 x c	1.644 x c	1.575 x c	1.480 x c	1.221 x c	0.889 x c	0.707 x c	0.527 x c	0.300 x c
<i>Calculated Headlight (ϕ')</i>	0.000 degrees	17.707 degrees	26.681 degrees	35.818 degrees	55.008 degrees	76.996 degrees	89.424 degrees	108.400 degrees	180.000 degrees
<i>Calculated Doppler Shift (f')</i>	1.700 x f	1.682 x f	1.659 x f	1.622 x f	1.490 x f	1.215 x f	1.010 x f	0.705 x f	0.300 x f

Table #1

As the calculated results, in the table above, demonstrate, the headlight effect, throughout the subluminal region, changes all initial angles of emitted light; except for light emitted exactly along the two angles: 0° and 180° , which remains along the same directions.

In the Reference Frame of the Stationary Observer:

In the reference frame of the stationary observer, the initial angle of emission ϕ cannot be observed; and only the angle of the headlight effect ϕ' is observable, and labeled as the angle θ between the direction of the incident light and the velocity vector of the light source.

Let's assume that v_s denotes the velocity of the emitting light source with respect to the reference frame of the stationary observer. And that θ denotes the angle made by the direction of incident light to the velocity vector of the light source, as measured in the reference frame of the same observer.

According to the Galilean theorem of velocity addition, therefore, the velocity resultant of incident light, c' , with respect to the observer's reference frame can be calculated through the use of the following equation:

$$c' = c \sqrt{1 - \frac{v_s^2}{c^2} \sin^2 \theta} + v_s \cos \theta \quad 1.A.4$$

where θ is the angle made by the incident light to the velocity vector of the moving light source, as measured in the stationary observer's reference frame; and hence, θ is the same angle of headlight effect ϕ' as computed in the reference of the moving light source

As for the Doppler effect, due to the motion of the light source, it can be calculated in the reference frame of the observer by using this equation:

$$f' = f \left(1 + \frac{v_s \cos \theta}{c \sqrt{1 - \frac{v_s^2}{c^2} \sin^2 \theta}} \right) \quad 1.A.5$$

where Equation #1.A.5 is the standard equation for computing the Doppler effect in the reference frame of the stationary observer.

Equation #1.A.3 and Equation #1.A.5 must always give the same numerical results for the Doppler effect, in all cases in which v_s has the same value and this condition is satisfied:

$$\phi' = \theta$$

Assuming that light is emitted isotropically in all directions, as observed in the reference frame of the moving light source, the above equations for computing the headlight effect and Doppler effect, within the subluminal region, in which:

$$0 \leq v < c$$

lead to the following conclusions:

- Light emitted in the forward direction, within a solid angle of 2π , with respect to the velocity vector of the emitting light source, is shifted by the headlight effect in the same forward direction to form a cone with a decreasing half-angle with the increasing velocity magnitude of the light source; i.e., the greater the velocity of the light source; the narrower the forward cone of shifted light; and vice versa. But since the velocity of the light source, in the subluminal region, is always less than c , the half-angle of the forward cone of shifted light remains always greater than an angle of 45° . And that is because the headlight effect on light emitted at right angles to the velocity vector of the light source defines effectively and determines always the half-angle values of the shifted forward cone; and because only at:

$$v_s = c$$

light emitted at right angles to the velocity vector of the light source is shifted, by the headlight effect, to an angle of 45° in the forward direction.

- Light emitted in the forward direction, within a 2π -solid angle, with respect to the velocity vector of the emitting source, is Doppler-blue shifted due to the motion of the light source.
- Light shifted by the headlight effect, in the forward direction, with respect to the velocity vector of the emitting source, has different velocity values within the superluminal range:

$$c < c' < 2c$$

depending on the initial direction of emission.

- The intensity of all of the light emitted in the forward direction, within a solid angle of 2π , with respect to the velocity vector of the emitting source, and shifted by the headlight effect in the same forward direction to form a narrow cone, in addition to its increase by the Doppler effect and the superluminal speeds of light, is also boosted to higher levels by the headlight effect, in accordance with this relation:

$$I' = I \left[\frac{\phi}{\arcsin\left(\frac{c}{c'} \sin \phi\right)} \right]$$

where I is the initial intensity; I' is the observed intensity in the reference frame of the stationary observer; and ϕ is the initial angle of emission, as measured in the reference frame of the moving light source.

- Light emitted in the backward direction, within a 2π -solid angle, with respect to the velocity vector of the emitting source, is shifted by the headlight effect towards the forward direction to form a wider cone with an increasing half-angle with the increasing velocity of the light source.
- Light emitted in the backward direction and remains, within a solid angle of 2π in the same direction, with respect to the velocity vector of the emitting source, is red shifted by the Doppler effect due to the subluminal motion of the light source.
- Light emitted initially, within a backward solid angle of 2π , and then shifted by the headlight

effect to form a forward cone, with respect to the velocity vector of the emitting source, is Doppler blue shifted due to the motion of the light source.

- Light emitted in the backward direction and remains, within a solid angle of 2π in the same direction, with respect to the velocity vector of the emitting source, has velocity values within the subluminal range:

$$0 \leq c' \leq c\sqrt{1 - v_s^2/c^2}$$

depending on the initial direction of emission.

- Light emitted initially, within a backward solid angle of 2π , and then shifted by the headlight effect to form a forward light cone, with respect to the velocity vector of the emitting source, has, for all velocity values of the light source in this range:

$$v_s < c$$

different velocity values within the subluminal-superluminal range:

$$c\sqrt{1 - v_s^2/c^2} \leq c' \leq c\sqrt{1 + v_s^2/c^2}$$

depending on the initial direction of emission.

- The intensity of all light emitted in the backward direction and remains, within a solid angle of 2π in the same direction, with respect to the velocity vector of the emitting source, in addition to its decrease by the Doppler effect and the subluminal speeds of light, is decreased to lower levels by the headlight, in accordance with this relation:

$$I' = I \left[\frac{\arcsin\left(\frac{c}{c'} \sin \phi\right)}{\phi} \right]$$

where I is the initial intensity; I' is the observed intensity in the reference frame of the stationary observer; and ϕ is the initial angle of emission, as measured in the reference frame of the moving light source.

B. Light Aberration in the Subluminal Region:

Light aberration, by definition, is the bending of incident light in the forward direction, due to the motion of the receiving observer.

On the basis of ballistic and emission theories, light aberration is, quite simply, the direction of the relative velocity resultant of the velocity of incident light and the velocity of the observer, as measured in the observer's frame of reference.

There are only two cases to be considered with regard to light aberration in the subluminal region:

The Case of Incident Light from a Stationary Light Source:

In this case, light aberration is obtained by using the following standard equation:

$$\sin \Delta\beta = \frac{v_o}{c} \sin \beta \quad 1.B.1$$

where v_o is the velocity of the observer; and β is the apparent position of the light source with respect to the velocity vector of the observer.

And accordingly, the Doppler effect, due to the motion of the observer, can be calculated in the reference frame of the observer by using this equation:

$$f' = f \left(1 + \frac{v_o \cos \beta'}{c} \right) \quad 1.B.2$$

where f' is observed frequency; f is the emitted frequency; and β' is the true position of the light source, at the time of emission, with respect to the velocity vector of the moving observer:

$$\beta' = \beta + \Delta\beta$$

and where $\Delta\beta$ is computed by using Equation #1.B.1.

And also, to obtain the relative velocity resultant of light c' , the same angle β' defined above can be inserted as well into this equation:

$$c' = c \sqrt{1 + \frac{v_o^2}{c^2} + 2 \frac{v_o}{c} \cos \beta'}$$

for computing the relative velocity resultant of incident light in the reference frame of the moving observer.

It should be noted, within this context, that the main difference between the relative velocity resultant of incident light, calculated by using the above equation, and the ballistic velocity resultant of emitted light, computed by using Equation #1.A.1, is that the former is observed only in the reference frame of the moving observer; while the latter is the actual velocity resultant at which light emitted, by the moving light source in a specific direction, travels in vacuum.

The Case of Incident Light from a Moving Light Source:

In this particular case, the light source and the observer are in motion; and light aberration is obtained by calculating the direction of the relative resultant of the velocity of the observer v_o and the velocity of incident c' as given by Equation #1.A.4, in accordance with this equation:

$$\sin \Delta\beta = \frac{v_o}{c'} \sin \beta = \frac{v_o \sin \beta}{c \sqrt{1 - \frac{v_s^2}{c^2} \sin^2 \theta + v_s \cos \theta}} \quad 1.B.3$$

where v_o is the velocity of the observer; and v_s is the velocity of the light source.

And consequently, the Doppler effect, due to the motion of the observer and the light source, can be calculated in the reference frame of the moving observer by using this equation:

$$f' = f \left(1 + \frac{v_o \cos \beta' + v_s \cos \theta}{c \sqrt{1 - \frac{v_s^2}{c^2} \sin^2 \theta}} \right) \quad 1.B.4$$

where f' is observed frequency; f is the emitted frequency; and β' is the true position of the light

source, at the time of emission, with respect to the velocity vector of the moving observer:

$$\beta' = \beta + \Delta\beta$$

and where $\Delta\beta$ is computed by using Equation #1.B.3.

On the assumption that all of the incident light is received from all directions by the moving observer, as in the case of the cosmic background radiation [Ref. #8], for example, the above equations for computing light aberration, in the subluminal region, in which:

$$0 \leq v_o < c$$

lead to the following conclusions:

- All of the incident light from the forward direction, within a 2π -solid angle, with respect to the velocity vector of the receiving observer, is shifted by light aberration in the same forward direction to form a narrow cone of a half-angle that decreases with the increasing velocity of the receiving observer. But since the velocity of the observer, in the subluminal region, does not exceed the value of c , shifted light remains always within a cone of a half-angle greater than 45° ; and has relative velocities within the superluminal range:

$$c < c' < 2c$$

depending on the initial angle of incidence.

- All of the incident light from the forward direction, within a solid angle of 2π , with respect to the velocity vector of the receiving observer, is Doppler blue shifted by the motion of the observer.
- The intensity of incident light from the forward direction, within a 2π -solid angle, with respect to the velocity vector of the receiving observer, and shifted by light aberration in the same forward direction to form a narrow cone, in addition to its increase by the Doppler effect and the superluminal relative speeds of light is also boosted to higher levels by the light aberration, in accordance with this relation:

$$I' = I \left(1 + \frac{\Delta\beta}{\beta} \right)$$

where I is the initial intensity; I' is the observed intensity in the reference frame of the moving observer; and β is the apparent position of the light source with respect to the velocity vector of the receiving observer.

- All of the incident light from the backward direction, within a solid angle of 2π , with respect to the velocity vector of the receiving observer, is shifted by light aberration towards the forward direction to form a wider cone with a half-angle that increases with the increasing velocity of the observer.
- All of the incident light from the backward direction and remains, within a 2π -solid angle in the same direction, with respect to the velocity vector of the receiving observer, is Doppler red shifted by the motion of the observer.
- All of the incident light received initially, within a backward solid angle of 2π , and then shifted by light aberration to a forward cone, with respect to the velocity vector of the receiving observer, is Doppler blue shifted due to the motion of the observer.
- The intensity of incident light received from the backward direction and remains, within a 2π -solid angle in the same direction, with respect to the velocity vector of the receiving observer, in addition to its decrease by the Doppler effect and the subluminal relative speeds of light is decreased to lower levels by light aberration, in accordance with this relation:

$$I' = I \left(1 - \frac{\Delta\beta}{\beta} \right)$$

where I is the initial intensity; I' is the observed intensity in the reference frame of the moving observer; and β is the observed angle of incidence, as measured in the reference frame of the moving observer.

2. The Doppler Beaming in the Near Superluminal Region:

The near superluminal region is outlined by the following condition:

$$c \leq v \leq 2c$$

where v is the velocity of the light source or the moving observer; and c is the muzzle speed of light.

And therefore, the speed of the emitting light source and the receiving observer, v , in this region, does not exceed the value of $2c$.

A. The Headlight Effect in the Near Superluminal Region:

In the near superluminal region, emitted light, as observed in the reference frame of the moving light source, remains, always, isotropic and exactly the same as in the stationary case, regardless of the magnitude and the direction of the velocity vector of the emitting light source.

Nevertheless, it should be clear from Equation #1.A.2 :

$$\sin \phi' = \frac{\sin \phi}{\sqrt{1 + \frac{v_s^2}{c^2} + 2 \frac{v_s}{c} \cos \phi}}$$

for computing the headlight effect on the basis of ballistic and emission theories of light, in the reference frame of the moving light source, that, at the start of the near superluminal region; i.e., when:

$$v_s = c$$

light emitted in the backward direction, within a 2π -solid angle, is entirely shifted by the headlight effect to for a forward cone, with respect to the velocity vector of the emitting light source.

And therefore, from this point at the beginning of the near superluminal region, at c , and up to infinity, no light, emitted by the moving light source, can propagate or reach any observer in any direction, within a backward solid angle of 2π , with respect to the velocity vector of the emitting light source.

As in the case of the subluminal region, Equation #1.A.2 and Equation #1.A.3 are used, in this case as well to obtain the numerical values for the headlight effect and the Doppler effect, in the reference of the moving light source.

If it's assumed, for instance, that the velocity of the light source is:

$$v_s = 1.5c$$

then, the headlight effect and the Doppler effect for light, emitted in any number of directions, can be

computed by the above two equations, as in the following table:

<i>Initial Direction of Emission (ϕ)</i>	0.000 degrees	30.000 degrees	45.000 degrees	60.000 degrees	90.000 degrees	120.000 degrees	135.000 degrees	150.000 degree	180.000 degrees
<i>Calculated Speed of Light (c')</i>	2.500 x c	2.418 x c	2.318 x c	2.179 x c	1.803 x c	1.323 x c	1.062 x c	0.807 x c	0.500 x c
<i>Calculated Headlight (ϕ')</i>	0.000 degrees	11.934 degrees	17.764 degrees	23.413 degrees	33.690 degrees	40.893 degrees	41.727 degrees	38.262 degrees	0.000 degrees
<i>Calculated Doppler Shift (f')</i>	2.500 x f	2.544 x f	2.606 x f	2.715 x f	3.250 x f	4.096 x f	-2.998 x f	-2.176 x f	-0.500 x f

Table #2

From the calculated numerical results above, the following points can be made with regard to the general characteristics of shifted electromagnetic radiation in the near superluminal region:

I. Light emitted along the forward angle of 0° , in the reference frame of the moving light source, always remains along the same direction, as measured in the observer's frame of reference, regardless of the velocity magnitude of the light source.

II. Light emitted in the backward direction along an angle of 180° , in the reference frame of the moving light source, is shifted by the headlight effect to the opposite direction and received along a forward angle of 0° by the stationary observer.

III. Light traveling at a velocity resultant c' , where:

$$c' \leq v_s \cos \phi'$$

must undergo negative Doppler effect in the reversed direction to its initial direction of emission. For instance, light, emitted along the backward angle of 180° , is shifted towards the red by the Doppler effect, in all cases of:

$$c' < c$$

and towards the blue by the Doppler effect , in all cases of:

$$c' > c$$

while, in the special case of:

$$c' = c$$

it does not undergo any Doppler shift at all.

IV. Light traveling at a velocity resultant c' , where:

$$c' \leq v_s \cos \phi'$$

must be received, by the stationary observer, in reversed order to its initial sequence of emission; i.e., in the *first-emitted-last-received* order; instead of the familiar *first-emitted-first-received* sequence. And that is because, the light source, in this case, is traveling faster than and a head of the emitted light.

V. For values of the velocity of the light source v_s greater than c ; i.e.,

$$v_s > c$$

the role of the factor:

$$\left(1 - \frac{v_s^2}{c^2} \sin^2 \theta\right)^{-1/2}$$

in Equation #1.A.5:

$$f' = f \left(1 + \frac{v_s \cos \theta}{c \sqrt{1 - \frac{v_s^2}{c^2} \sin^2 \theta}} \right)$$

becomes significant in boosting the values of the Doppler effects for light emitted by the light source in all directions, except in the direction of 0° and the direction of 180° .

B. Light Aberration in the Near Superluminal Region:

As in the subluminal region, light aberration can be defined, in the near superluminal region, as the bending of incident light in the forward direction, due to the motion of the receiving observer.

And only two cases need to be investigated, here, with regard to light aberration:

The Case of Incident Light from a Stationary Light Source:

In this case, the emitting light source is stationary; and the velocity of the receiving observer, v_o , is within the following range:

$$c \leq v_o \leq 2c$$

And accordingly, light aberration, in this case, is computed by using Equation #1.B.1:

$$\sin \Delta\beta = \frac{v_o}{c} \sin \beta$$

where β is the apparent position of the light source.

Because, in the near superluminal region, the velocity of light from the stationary light source is less than velocity of the observer, v_o , for values of:

$$\beta > 90^\circ$$

no light from any stationary light source located anywhere within the backward solid angle of 2π , with respect to the velocity vector of the observer, can be received by the fast moving observer.

By contrast, the apparent position of any stationary light source, located anywhere within the forward solid angle of 2π , is shifted, in the forward direction, by light aberration to be within a narrow cone with a half-angle in the range of:

$$\beta \leq 45^\circ$$

with respect to the velocity vector of the observer.

The Case of Incident Light from a Moving Light Source:

In this case, the observer and the light source are in motion; and light aberration is obtained by using Equation #1.B.3:

$$\sin \Delta\beta = \frac{v_o \sin \beta}{c \sqrt{1 - \frac{v_s^2}{c^2} \sin^2 \theta} + v_s \cos \theta}$$

where θ is the direction of emitted light with respect to the velocity vector of the light source; and β is the apparent position of the light source with respect to the velocity vector of the observer.

And so, light from any approaching light source located anywhere within the backward solid angle of 2π , with respect to the velocity vector of the observer, can be received by the moving observer in all cases, in which the following condition is true:

$$c' > v_o$$

where c' is the velocity of incident light; and v_o is the velocity of the observer.

In addition, the true position of any approaching light source, at the time of emission, located anywhere within the forward solid angle of 2π , with respect to the velocity vector of the observer, is shifted less; and the true position of any receding light source, within the same solid angle, is shifted more, by light aberration, than that of the stationary light source, in these two cases respectively.

3. The Doppler Beaming in the Far Superluminal Region:

The far superluminal region is defined according to the following condition:

$$2c < v < \infty$$

where v is the velocity of the light source or the moving observer; and c is the muzzle speed of light.

And therefore, the speed of the emitting light source and the moving observer, v , in the far superluminal region, is, always, greater than $2c$.

A. The Headlight Effect in the Far Superluminal Region:

The headlight effect, ϕ' , in the far superluminal region, is computed by using Equation #1.A.2:

$$\sin \phi' = \frac{\sin \phi}{\sqrt{1 + \frac{v_s^2}{c^2} + 2 \frac{v_s}{c} \cos \phi}}$$

where v_s is the velocity of the light source; and ϕ is the initial direction of emitted light in the reference frame of the moving light source.

And the Doppler effect is calculated by using Equation #1.A.3:

$$f' = f \left(1 + \frac{(v_s/c) \cos \phi'}{\sqrt{1 + \frac{v_s^2}{c^2} + 2 \frac{v_s}{c} \cos \phi} - (v_s/c) \cos \phi'} \right)$$

where f' is the observed frequency; and f is the emitted frequency.

Now, let's assume, for instance, that the velocity of the light source, v_s , is:

$$v_s = 150c$$

If it's assumed that light, as observed in the reference frame of the moving light source, is emitted isotropically in all directions, then the numerical results of the headlight effect and the Doppler effect, therefore, can be computed, for any number of directions, by using the above two equations, as summarized in the following table:

<i>Initial Direction of Emission (ϕ)</i>	0.000 degrees	30.000 degrees	45.000 degrees	60.000 degrees	90.000 degrees	120.000 degrees	135.000 degrees	150.000 degrees	180.000 degrees
<i>Calculated Speed of Light (c')</i>	151.000 x c	150.867 x c	150.709. x c	150.502 x c	150.003 x c	149.503 x c	149.295 x c	149.135 x c	149.000 x c
<i>Calculated Headlight (ϕ')</i>	0.000 degrees	0.190 degrees	0.269 degrees	0.330 degrees	0.382 degrees	0.332 degrees	0.271 degrees	0.192 degrees	0.000 degrees
<i>Calculated Doppler Shift (f')</i>	151.000 x f	173.845 x f	212.071 x f	298.329 x f	23685.10 x f	-302.342 x f	-212.273 x f	-172.58 x f	-149.000 x f

Table #3

From the calculated numerical results above, the following points can be made with regard to the primary characteristics of shifted electromagnetic radiation in the far superluminal region:

A. At the given velocity of the light source:

$$v_s = 150c$$

all of the light emitted along any direction, within the forward solid angle of 2π , as observed in the reference frame of the moving light source, is shifted further by the headlight effect to the forward direction, within an extremely narrow cone with a half-angle of 0.382° , as measured in the reference frame of the stationary observer.

B. At the given velocity of the light source:

$$v_s = 150c$$

light emitted along any direction, within the backward solid angle of 2π , as observed in the reference frame of the moving light source, is shifted by the headlight effect to an opposite direction, to form a narrow forward cone, as measured in the reference frame of the stationary observer.

C. Light emitted along any direction, within the forward solid angle of 2π , as observed in the reference frame of the moving light source, and shifted further by the headlight effect to the forward half-angle cone of 0.382° , as measured in the reference frame of the stationary observer, travels at superluminal velocities greater than the velocity of the light source; and hence, it travels always ahead of the moving light source.

D. Light emitted along any direction, within the backward solid angle of 2π , as observed in the reference frame of the moving light source, and shifted by the headlight effect to a narrow forward cone, as measured in the reference frame of the stationary observer, travels at superluminal velocities less than the velocity of the light source; and as a result, it travels always behind the fast moving light source.

E. Light emitted along any direction, within the forward solid angle of 2π , as observed in the reference frame of the moving light source, and shifted further by the headlight effect to a forward cone with a half-angle of 0.382° , as measured in the reference frame of the stationary observer, is shifted to positive blue values by the Doppler effect, and received in accordance with the *first-emitted-first-received* sequence by the stationary observer.

F. Light emitted along any direction, within the backward solid angle of 2π , as observed in the reference frame of the moving light source, and shifted by the headlight effect to a forward cone, as measured in the reference frame of the stationary observer, is shifted to negative blue values by the Doppler effect, and received in accordance with the *first-emitted-last-received* sequence by the stationary observer. And it should be noted, here, that the aforementioned blue Doppler shifts have negative signs with respect to the initial directions of the emitted light only; and that the negative Doppler blue shifts are the same as the positive Doppler blue shifts in every other respect.

G. Light emitted along the angle of 90° , as observed in the reference frame of the moving light source, and shifted by the headlight effect to the forward direction to a small angle of 0.382° , as measured in the reference frame of the stationary observer, has the largest Doppler blue shift f' :

$$f' = f[23685.10]$$

in Table #3.

And that is because the velocity resultant of light emitted at right angles, in this case, is very close to the velocity of the light moving source; i.e.,

$$c' - v_s \approx 0.003c$$

where c' is the velocity resultant of light emitted at right angles to the velocity vector of the light source; and v_s is the velocity of the emitting light source.

H. In the far superluminal region, the total energy of light emitted in any direction, as observed in the reference frame of the moving light source, is boosted by the Doppler effect, as measured in the reference frame of the stationary observer, in accordance with the following equation:

$$E' = E \left(\frac{f'}{f} \right) \left(\frac{c'}{c} \right)^2 = E \left(\frac{\left(1 + \frac{v_s^2}{c^2} + 2 \frac{v_s}{c} \cos \phi \right)^{\frac{3}{2}}}{\sqrt{1 + \frac{v_s^2}{c^2} + 2 \frac{v_s}{c} \cos \phi - (v_s/c) \cos \phi'}} \right)$$

where E' is the observed energy; E is the emitted energy; and ϕ is the initial direction of emission.

B. Light Aberration in the Far Superluminal Region:

Light aberration can be defined, in the far superluminal region, as the bending of incident light in the forward direction, due to the motion of the receiving observer.

There are only two cases that have to be considered, here, with regard to light aberration:

The Case of Incident Light from a Stationary Light Source:

In this case, the emitting light source is stationary; and the velocity of the receiving observer, v_o , is within this range:

$$2c < v_o < \infty$$

And accordingly, light aberration is computed by using Equation #1.B.1:

$$\sin \Delta\beta = \frac{v_o}{c} \sin \beta$$

where β is the apparent position of the light source.

Because, in the far superluminal region, the velocity of light from the stationary light source is less than the velocity of the observer, v_o , for all values of:

$$\beta > 90^\circ$$

no light from any stationary light source located anywhere within the backward solid angle of 2π , with respect to the velocity vector of the observer, can be received by the fast moving observer.

By contrast, the apparent position of any stationary light source, located anywhere within the forward solid angle of 2π , is shifted, in the forward direction, by light aberration to be within a narrow light cone with a half-angle in the range of:

$$\beta \leq 26.565^\circ$$

with respect to the velocity vector of the observer.

The Case of Incident Light from a Moving Light Source:

In this case, the light source is in motion; and light aberration is obtained by using Equation #1.B.3:

$$\sin \Delta\beta = \frac{v_o \sin \beta}{c \sqrt{1 - \frac{v_s^2}{c^2} \sin^2 \theta + v_s \cos \theta}}$$

where θ is the direction of incident light with respect to the velocity vector of the light source; and β is the apparent position of the light source with respect to the velocity vector of the observer.

And accordingly, light from any approaching light source located anywhere within the backward solid angle of 2π , with respect to the velocity vector of the observer, can be received by the moving observer in all cases, in which the following condition is fulfilled:

$$c' > v_o$$

where c' is the velocity resultant of incident light; and v_o is the velocity of the observer.

In addition, the true position of any approaching light source, at the time of emission, located anywhere within the forward solid angle of 2π , with respect to the velocity vector of the observer, is shifted less; and the true position of any receding light source, within the same solid angle, is shifted more, by light aberration, than that of the stationary light source, in the two cases respectively.

4. Concluding Remarks:

Since Doppler beaming implies, by definition, the bending of emitted and incident light rays in the forward direction, due to the motion of the emitting light source, the motion of the receiving observer, or both; one of the most relevant questions, within the current context, ought to be this:

Is it mathematically possible to obtain some or all of the same exact numerical results by treating light aberration as headlight effect; and vice versa?

In other words, are the headlight effect and light aberration partially or totally symmetrical?

Clearly, the relative velocity, in the two cases, is symmetrical; i.e., it's equally possible to assume that the observer is at rest, and the light source is in motion; or to assume that the light source is at rest, and the observer is in motion.

Moreover, the stationary light source appears, at all times, as seen in the moving observer's reference frame, to be moving in the opposite direction at exactly the same speed as that of the moving observer.

And so, are the headlight light effect and light aberration completely symmetrical in the same way?

To answer this question, let's assign the actual motion of the observer to the stationary light source, and treat the aforementioned apparent velocity as the actual velocity of the light source.

If the velocity of the observer is assigned to the light source, then the value of the apparent position of the light source β , relative to the velocity vector of the observer, must be assigned to the angle θ made by the direction of the relative velocity resultant of light to the apparent velocity vector of the light source, as measured in the reference frame of the observer.

And since the angle θ is the same angle labeled as ϕ' , in the light source's frame of reference, it's immediately clear that the numerical magnitude of the ballistic velocity resultant of light c'_B :

$$c'_B = c \sqrt{1 + \frac{v_s^2}{c^2} + 2 \frac{v_s}{c} \cos \phi}$$

and the numerical magnitude of the relative velocity resultant of light c'_R :

$$c'_R = c \sqrt{1 + \frac{v_o^2}{c^2} + 2 \frac{v_o}{c} \cos \beta'}$$

in these two cases, remain the same and symmetrical; whether light aberration is treated as headlight effect; or whether the opposite is true.

And that is because, if it's assumed that:

$$\phi' = \theta = \beta$$

then it must follow that:

$$\phi = \beta'$$

and the two angles have the same value in the equation for computing the ballistic velocity resultant of light and the equation for calculating the relative velocity resultant of light, in the two cases respectively.

And also, although the true position of the light source, at the time of emission, in the case of light aberration is not the same position as in the case of the headlight effect, its instantaneous position, at the time of reception, remains unchanged and the same, regardless of whether it's assumed that the observer is at rest, and the light source is in motion; or vice versa.

And therefore, only the Doppler effect needs to be investigated, here, in detail, in order to determine whether it's symmetrical, in the two cases under discussion; or it's, in fact, asymmetrical.

To calculate the Doppler effect, f' , due to the motion of the light source, we insert the assigned velocity value above into Equation #1.A.5:

$$f' = f \left(1 + \frac{v_s \cos \theta}{c \sqrt{1 - \frac{v_s^2}{c^2} \sin^2 \theta}} \right)$$

which is the standard equation for computing the Doppler effect in the reference frame of the stationary observer.

Next, we insert the same assigned value into Equation #1.B.2:

$$f' = f \left(1 + \frac{v_o \cos \beta'}{c} \right)$$

where f'' is observed frequency; f is the emitted frequency; and β' is the true position of the light source with respect to the velocity vector of the moving observer:

$$\beta' = \beta + \Delta\beta$$

and where $\Delta\beta$ is computed by using Equation #1.B.1.

By comparing the numerical results obtained from the two equations above, we conclude that since:

$$\cos \theta > \cos \beta'$$

and because Equation #1.A.5 contains the boosting factor:

$$\left(1 - \frac{v_s^2}{c^2} \sin^2 \theta \right)^{-1/2}$$

that, for any given values of the relative velocity between the observer and the light source, the numerical values of the Doppler effect, f' , obtained from Equation #1.A.5 are always greater than the numerical values obtained from Equation #1.B.2.

And therefore, even though the relative velocity resultant, the ballistic velocity resultant, and the instantaneous position of the light source at the time of reception, are symmetrical, in the two cases respectively, the Doppler effect is distinctly asymmetrical; and its values are greater in the case of the headlight effect than its values in the case of light aberration. And as a result, the headlight effect and light aberration are partially symmetrical.

In addition to producing measurably smaller values for the Doppler effect, light aberration cannot, under any conceivable circumstances, recreate the remarkable light cones of the headlight effect.

The two cones of light — one traveling ahead of the moving light source, and one traveling behind it — are among the most striking characteristics of shifted electromagnetic radiation by the headlight effect, in the far superluminal region, in any treatment based upon ballistic and emission theories.

Although there is a speed difference of $2c$ between the shifted light at the center of the preceding cone and the shifted light at the center of the trailing cone, the difference between the two cones, in terms of total radiation energy and light travel time, approaches zero, as the velocity of the light source approaches infinity.

In the far superluminal region, light shifted by the headlight effect, inside each of these two narrow cones, has all the unique properties and the salient characteristics of the well-known synchrotron radiation.

However, there is one major difference between the synchrotron radiation, as treated in accordance with the current conventional theories; and the same synchrotron radiation, as treated within the framework of ballistic and emission theories: According to the former type of theories, all light shifted, by the headlight effect, travels at the same speed of c . While, by contrast, according to the latter kind of theories, the shifted light of the faster preceding cone travels at various speeds within the superluminal range of:

$$c \left(1 + \frac{v_s^2}{c^2} \right)^{1/2} \leq c' \leq (c + v_s)$$

and the shifted light of the slower trailing cone travels at speeds within this superluminal range:

$$(c - v_s) \leq c' < c \left(1 + \frac{v_s^2}{c^2} \right)^{1/2}$$

where c' is the velocity resultant of light; and v_s is the velocity of the light source.

According to ballistic and emission theories, therefore, short pulses of synchrotron radiation must become longer and longer along the direction of propagation, as they travel farther and farther from the emitting light source, due to the above differences in their speeds .

There is also another important difference between the two categories of theories with regard to the physical mechanisms behind the phenomenon of synchrotron radiation as observed in laboratory settings and astronomical settings.

Within the framework of current conventional theories, Doppler beaming is the only physical mechanism behind the terrestrial synchrotron and the astronomical synchrotron as well.

On the basis of ballistic and emission theories, however, there is an additional physical mechanism, which in astronomical settings can produce synchrotron radiation:

If an approaching light source accelerates, or a receding light source decelerates, during the time of emission, then, according to ballistic and emission theories, the part of light emitted, at the end of the wave period, must travel, by a small but measurable amount, faster than the part of light emitted, at the start of the same wave period. And so, when the faster part starts to overtake the slower part, the frequency of light increases; and eventually the emitted light becomes synchrotron radiation as

measured by observers at vast distances from the emitting light source.

Because of its much lower energy requirement than that of Doppler beaming, this second physical mechanism is far more likely to be behind the observed synchrotron radiation in most astronomical settings inside the Milky Way as well as outside of it.

Finally, it should be pointed out that the headlight effect, on electromagnetic radiation from moving light sources, can make calculations, based upon the Galilean transformation within the context of ballistic and emission theories, much more complicated than expected at first glance, and lead to surprising numerical predictions in a fairly significant number of situations.

One of the least expected predictions, in this regard, is the conservation of the light travel time, according to which the flight times of rays of light, when the light source is stationary, and the flight times of the same rays of light, shifted by the headlight effect, over all displacements at right angles to the velocity vector of the light source, when the light source is in motion, remain exactly the same and equal, regardless of the speed of the light source.

Nonetheless, when the velocity resultant of light c' is given in its Cartesian form:

$$(c'_x, c'_y) = ([c \cos \phi + v_s], c \sin \phi)$$

the above prediction should become intuitive and , at once, obvious; since the vertical component ($c \sin \phi$) of the velocity resultant of light remains unchanged and constant regardless of the values of the velocity of the light source v_s .

To demonstrate that is indeed the case, let's consider, for instance, a straight line parallel to the velocity vector of the moving light source, as illustrated below:

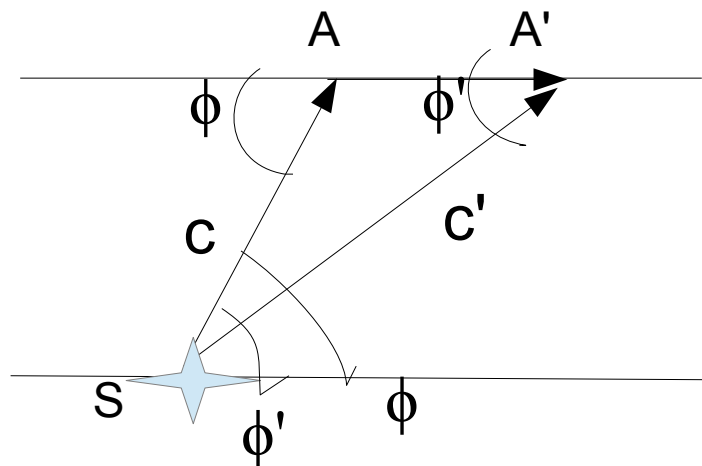


Illustration: Headlight Effect

When the light source is at rest, an emitted ray of light that meets the parallel straight line at Point A located at a distance d from the stationary light source, has a travel time of T :

$$T = \frac{d}{c}$$

where c is the muzzle speed of light.

And likewise, when the light source is in motion, the same ray of light traveling at the velocity resultant of c' , must be shifted by the headlight effect to meet the same parallel straight line at Point A' located at a distance d' from the location of the moving light source at the time emission; and hence:

$$T' = \frac{d'}{c'}$$

where T' is the travel time of the shifted ray of light from its emission location to Point A' .

By applying the law of sines to the above illustration, we obtain the following equation:

$$\frac{c'T'}{\sin \phi} = \frac{cT}{\sin \phi'}$$

where ϕ is the direction of a ray of light emitted by a stationary light source; and ϕ' is the shifted direction of the same ray of light, due to the headlight effect caused by the motion of the light source.

And hence, by inserting the value of c' from Equation Equation #1.A.2, and the value of ϕ' from Equation #1.A.2, into the above equation, we obtain the following result:

$$T' = T \frac{c}{c'} \left(\frac{\sin \phi}{\sin \phi'} \right) = T$$

It follows, therefore, that the numerical values of light travel time, over all displacements at right angles to the velocity vector of the light source, are conserved and unchanged by the motion of the light source.

REFERENCES:

1. **MathPages:**
["Stellar Aberration"](#)
2. **Bradley's Discovery of Stellar Aberration:**
["The Search for Proof That the Earth Moves"](#)
3. **Stellar Aberration**
["The Contradiction between Einstein and Bradley"](#)
4. **Newton Physics:**
["Stellar Aberration and Einstein's Relativity"](#)
5. **Bright Lights:**
["The Headlight Effect"](#)
6. **Wikipedia:**
["Relativistic beaming"](#)
7. **The Astrophysical Journal:**
["Observations of Doppler Boosting in Kepler Light Curves"](#)
8. **RSSD - ESA:**
["Doppler boosting the CMB"](#)
9. **Stewart, O. M.,**
(1911). "The Second Postulate of Relativity and the Electromagnetic Emission Theory of Light",
Phys. Rev. 32: 418-428.
10. **Ritz, W.,**
["The Role of Aether in Physics"](#).

Related Papers:

- A. **Effect of Reflection from Revolving Mirrors on the Speed of Light:**
[A Brief Review of Michelson's 1913 Experiment](#)
- B. **Doppler Effect on Light Reflected from Revolving Mirrors:**

A Brief Review of Majorana's 1918 Experiment

- C.** ***Absolute Velocities:***
 The Detailed Predictions of the Emission Theory of Light
- D.** ***The LLR Experiment:***
 An Analysis of the Lunar Laser Ranging Test of the Invariance of c
- E.** ***The Double-Star Experiment:***
 A Comprehensive Review of de Sitter's 1913 Demonstration