the general science journal

The Coulomb Electron Speed and Acceleration in Bohr's Model of the Hydrogen Atom

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Coulomb's law defines the electrical force between two charged objects which is directly proportional to the product of the charges on these two objects and inversely proportional to the square of the separation distance between them. In the equation form

$$F_C = k_e q_1 q_2/R^2 \quad \ldots (1)$$

where k_e is the Coulomb constant (= 8,988 × 10^9 $N\ m^2\ C^{-2}$)[1], q_1 is the charge of the first object, q_2 is the charge of another object and R is the distance between these objects.

We will apply the expression (1) in the case of the proton and electron of the hydrogen atom. The proton has a positive charge equal in magnitude to a unit of electron charge e (= 1.60 × 10^{-19} *C*). In the ground state of hydrogen atom, its Bohr radius $a_0 = 5.3 \times 10^{-11}$ *m*. The rest mass of electron $m_e = 9.11 \times 10^{-31}$ *kg*.

The Coulombic force F_p between these two charges is at a distance R would be $F_p = k_e e^2/R^2$. The Coulomb force of hydrogen atom is attractive and can cause the electron to deflect from a straight-line path to a circular path (Bohr's orbit) around the proton orbiting with a speed υ_e.

We know in this orbit the electron is exposed to the Coulombic centripetal force: $F_p = k_e e^2/R^2$ and the centrifugal force: $F_f = m_e \upsilon_n^2/R$. These two forces are balanced: $F_p = F_f$ or $k_e e^2/R^2 = m_e \upsilon_n^2/R$. An electron to move one-dimensionally along the straight line towards the proton Fc > Fp or, after some algebra, $(m_e \upsilon_n R)\upsilon_n < k_e e^2$. Applying Bohr's quantum condition $m_e \upsilon_n R = n\hbar$[2], after a bit of algebra, we get

$$\upsilon_n < (k_e e^2/\hbar) \times 1/n.$$

Let us assume that the electron moves along a straight one-dimensional path in the direction of the proton of hydrogen atom with a final speed equal to υ_1 about 2.2 × 10^6 m sec^{-1}. When this electron reaches the ground state it would orbit the proton in a circle with this speed as in Bohr's model.

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[1] To avoid confusion in further text, the SI units are given in italics.

[2] We know that $m_e \upsilon_n R$ is the angular momentum of an electron in its orbit which is, according to Bohr's model, quantized.

According to Newton's second law of motion, the acceleration of the electron at R would be equal to the Coulombic force divided by the mass of electron m_e. This can be represented by the equation

$$a_C = (F_C/m_e) = k_e e^2/m_e R^2 \quad \ldots (2).$$

By employing eqn. (2), we find that an electron situated at a distance of Bohr's radius from the proton would reach an acceleration of about 10^{30} *m sec*$^{-2}$. Taking it - as an average acceleration - the electron would reach the speed υ_1 for $\Delta t \approx 2.2 \times 10^{-24}$ sec.

To get as close to the proton as possible the electron must move at a relativistic speed much higher than υ_1 or close to the speed of light c ($\approx 3 \times 10^8$ m sec^{-1}). The relativistic centrifugal force $F_f = m_e\upsilon_e^2/R_{min} /[\sqrt{(1 - \upsilon_e^2/c^2)}]$ has to be equal to the centripetal force $F_p = k_e e^2/R_{min}$. This can be expressed as the equation

$$m_e\upsilon_e^2/R_{min}\sqrt{(1 - \upsilon_e^2/c^2)} = k_e e^2/R_{min}^2$$

or, after some algebra,

$$R_{min} = (k_e e^2/m_e) \times [\sqrt{(1 - \upsilon_e^2/c^2)}] \times 1/\upsilon_e^2$$

where R_{min} is the smallest distance between the electron approaching the proton with a relativistic speed υ_e.

Solving for R_{min} and substituting known values, we get

$$R_{min} \approx 2.6 \times 10^{-7} [\sqrt{(1 - \upsilon_e^2/c^2)}] \times 1/\upsilon_e^2.$$

The electron is traveling at 99.9999992 % of the speed of light[3] (or $\upsilon_e = 0.999999992c$) in the electron beam of energy as high as 4 GeV.[4] Plugging this value for υ_e after some calculation we find that R_{min} would be about 3.6×10^{-28} m. For comparison, the charge radius of proton is about 1×10^{-15} *m*.

Let us denote with a_e the average acceleration of the electron approaching to the proton. This acceleration would be higher than a_C and it can be expressed by the following equation

$$a_e = (\upsilon_e - \upsilon_1)/\Delta t$$

or

$$\Delta t = (\upsilon_e - \upsilon_1)/a_e$$

where Δt is the corresponding time interval. As $a_e > 10^{30}$ m sec^{-2}, we write $\Delta t < (\upsilon_e - \upsilon_1)/10^{30}$. Solving for Δt and substituting the known values we find $\Delta t < 3 \times 10^{-22}$ sec. Of note, it is generally accepted that the Planck time t_p ($\approx 10^{-43}$ *sec*) is the smallest time interval that has a physical

[3] Data from Jefferson Lab.

[4] The relativistic mass of the electron would be about 8000 m_e.

meaning. This time interval is about 10^{22} times smaller than the smallest time interval measured to date: 10^{-21} *sec*.